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Lattice B-parameters for $\Delta S = 2$ and $\Delta I = 3/2$ operators

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Abstract

We compute several matrix elements of dimension-six four-fermion operators and extract their B-parameters. The calculations have been performed with the tree-level Clover action at $\beta = 6.0$. The renormalization constants and mixing coefficients of the lattice operators have been obtained non-perturbatively. In the $\overline{\rm MS}$ renormalization scheme, at a renormalization scale $\mu \simeq 2$ GeV, we find $B_K(B_9^{3/2}) = 0.66(11)$, $B_7^{3/2} = 0.72(5)$ and $B_8^{3/2} = 1.03(3)$. The result for $B_8^{3/2}$ has important implications for the calculation of ϵ'/ϵ. © 1998 Elsevier Science B.V.

1. Introduction

The lattice evaluation of matrix elements of operators between hadronic states is a necessary ingredient to the calculation, from first principles, of a wide class of physical observables. In the effective Hamiltonian approach, weak amplitudes are expressed in terms of perturbative Wilson coefficients multiplied by matrix elements of local operators, which can be evaluated on the lattice. The results are conventionally presented in terms of B-parameters, which measure the deviation of the matrix elements from their value in the Vacuum Saturation Approximation (VSA). These quantities are subject to significant QCD corrections.

In this paper we focus on $\Delta S = 2$ and $\Delta I = 3/2$ transition amplitudes. The former case is characteristic of K^0–$\bar{K}^0$ oscillations, which are related to indirect CP violation, parametrized by ϵ. We compute B_K, the B-parameter of the matrix element $\langle \bar{K}^0 | O^{\Delta S=2} | K^0 \rangle$, where the dimension-six, four-fermion operator $O^{\Delta S=2}$ has a ''left-left'' chiral structure (see Section 2). We also compute the $\Delta I = 3/2$ amplitudes relevant in $K \to \pi\pi$ decays. These amplitudes are essential for theoretical predictions of direct CP-violation, parametrized by ϵ'. Matrix elements of the form $\langle \pi\pi | O^{3/2} | K \rangle$ enter in the calculation of ϵ', with two of the electro-penguin operators ($O_7^{3/2}$ and $O_8^{3/2}$) having a ''left-right'' chiral structure, and one operator ($O_9^{3/2}$) having a ''left-left'' one. Using Chiral Perturbation Theory (CPTh), these matrix elements can be related to the single-state matrix elements $\langle \pi^+ | O^{3/2} | K^+ \rangle$. In this work we compute the latter, parametrized in terms of B-parameters denoted as $B_7^{3/2}$ and $B_8^{3/2}$ (the conventional basis of operators for the $\Delta S = 1$ effective Hamiltonian can be found in Refs. [1]; we also remind the reader that $B_9^{3/2} = B_K$).

The Wilson lattice regularization breaks chiral symmetry. This implies that the $\Delta S = 2$ operator,

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which would otherwise renormalize multiplicatively, mixes with operators belonging to different chiral representations [2,3]. The same is true for the two $\Delta I = 3/2$ operators, which would otherwise only mix with each other, as they belong to the same chiral representation. Because of the mixing induced by the lattice, the correct chiral behaviour of the operators is achieved with Wilson fermions only in the continuum limit. For the $\Delta S = 2$ case, for example, restoration of the chiral properties amounts to the vanishing of the matrix element as $m_K \to 0$. In practice, the mixing of the $\Delta S = 2$ operator with operators of ''wrong'' naïve chirality, computed at small but finite cutoff a^{-1}, spoils the expected chiral behaviour [4,5] [1]. This problem can be attributed to two sources of systematic error in the computation of the matrix elements, namely the determination of the mixing coefficients in one-loop perturbation theory, and the $\mathcal{O}(a)$ (a is the lattice spacing) discretization errors. Several methods have been proposed in order to improve the determination of the mixing coefficients. One of them consists in evaluating the renormalization constants computed in Standard Perturbation Theory (SPT) using an effective coupling [7] which should reduce higher order corrections; we refer to it as Boosted Perturbation Theory (BPT). Another is the Non-Perturbative Method (NPM) for the computation of the renormalization constants on quark and gluon external states, as proposed in Ref. [8]. Finally, in the spirit of Ref. [3], the lattice mixing coefficients can also be obtained non-perturbatively by using the Ward Identity Method (WI's), with external quark and gluon states [9]; see also Refs. [10]. On the other hand, recent studies [11–13] (see also the present work) have shown that reducing the discretization error by using the tree-level Clover action does not improve the chiral behaviour of the matrix elements, even when BPT is implemented in the definition of the renormalized $\Delta S = 2$ operator. Instead, a good chiral behaviour has been observed by evaluating the renormalization constants non-perturbatively, either with the NPM (Refs. [11–13] and this work) or by using WI's (Ref. [9]). The fact that the restoration of the correct chiral behaviour has been seen both with the tree-level Clover action (Refs. [11–13] and the present work) and with the Wilson action (Ref. [9]), suggests that discretization effects are less important than those due to the perturbative evaluation of the mixing coefficients. Therefore, the recent Wilson fermion estimates of B_K from the NPM or the WI's, are considerably more reliable than those of earlier studies, based on perturbative renormalization.

Given the success of the NPM in the computation of B_K, we also apply it to the evaluation of the two $\Delta I = 3/2$ B-parameters of the electro-penguin operators, $B_7^{3/2}$ and $B_8^{3/2}$. A recent calculation, using the Wilson action and with the renormalization constants obtained in BPT, found $B_8^{3/2} = 0.81(3)$ [14], as opposed to the earlier results $B_8^{3/2} \simeq 1$ of Refs. [15,16]. In the present work, using the Clover action and the NPM for the evaluation of the renormalization constants we find $B_8^{3/2} = 1.03(3)$. Notice that, although obtained with a different lattice fermion action, our BPT estimate $B_8^{3/2} = 0.83(2)$ is, instead, fully compatible with that of Ref. [14]. Our preferred value $B_8^{3/2} = 1.03(3)$, obtained with an improved operator renormalized non-perturbatively, only suffers from discretization errors which are $\mathcal{O}(g_0^2 a)$. We thus believe that it is more reliable than previous results which have been obtained with the (non-improved) operator renormalized in one-loop perturbation theory. The same situation characterizes $B_7^{3/2}$: our NPM and BPT results are not in agreement, but the latter is compatible with the value quoted in [14]. We stress that a precise determination of $B_8^{3/2}$, combined with an equally reliable estimate of the strange quark mass, is essential to the determination of the ratio ϵ'/ϵ. For example, the uncertainties in the measurement of $B_8^{3/2}$, combined with the controversial results for m_s [17,18] may change ϵ'/ϵ up to a factor of 2 to 3. In view of its importance, we believe that $B_8^{3/2}$ should also be computed by applying the NPM with the Wilson action also and/or by using WI's for the determination of the lattice mixing coefficients of the renormalized operator.

An extensive study of the renormalization properties of the four-fermion operators can be found in [19]. There we detail all the theoretical and numeri-

[1] In the Staggered fermion approach, where chiral symmetry is partially preserved, the $\Delta S = 2$ matrix element displays the correct chiral behaviour. Thus, the B_K-parameter obtained with staggered fermions [6] has been deemed more reliable.

cal issues of relevance to the non-perturbative renormalization of the $\Delta S=2$ and $\Delta I=3/2$ operators. We have used these results in the present study.

The paper is organized as follows: in Section 2 we introduce the operators of interest and review briefly their phenomenological implications; in Section 3 we address the problem of operator mixing and give a brief account of the NPM for the computation of the renormalization constants; in Section 4 define the *B*-parameters and discuss their extraction from lattice correlation functions; in Section 5, we present our results for B_K, $B_7^{3/2}$ and $B_8^{3/2}$; finally, in Section 6 we compare our results to those previously obtained in the literature.

2. $\Delta s=2$ and $\Delta I=3/2$ transitions

Schematically, the $\Delta S=2$ effective Hamiltonian has the form

$$\mathscr{H}^{\Delta S=2}=\frac{G_F^2}{16\pi^2}M_W^2\,\hat{O}^{\Delta S=2}(\mu)\,\Phi(x_c,x_t,\mu) \qquad (1)$$

where $x_q=m_q^2/M_W^2$ and Φ is a combination of the Inami-Lim functions [20] weighted by the CKM matrix elements. All perturbative QCD corrections (known at the next-to-leading order (NLO) [21]) are included in Φ (which is, hence, μ-dependent). The rest of the notation is standard: G_F is the weak Fermi coupling, M_W the mass of the W boson and μ the renormalization scale. The $\Delta S=2$ operator in the above expression is the renormalized one. It is defined as follows:

$$O^{\Delta S=2}=\bar{s}\gamma_\mu^L d\bar{s}\gamma_\mu^L d, \qquad (2)$$

where $\gamma_\mu^L=\gamma_\mu(1-\gamma_5)$. The CP-violation parameter ϵ_K is defined as:

$$|\epsilon_K|=\frac{G_F^2}{16\pi^2}\frac{M_W^2}{\sqrt{2}\,\Delta m_K}\times\left(\tfrac{8}{3}f_K^2 m_K B_K(\mu)\right)\Phi(x_c,x_t,\mu), \qquad (3)$$

where Δm_K is the K_L^0-K_S^0 mass splitting. m_K and f_K denote the *K*-meson mass and decay constant, respectively.

In the $\Delta S=1$ case, the $\Delta I=3/2$ contribution to ϵ' can be written as

$$\mathrm{Im}\,\mathscr{A}^{3/2}\propto -G_F\,\mathrm{Im}\,\lambda_t\Big[C_7\langle O_7^{3/2}\rangle_{\mathrm{VSA}}B_7^{3/2}+C_8\langle O_8^{3/2}\rangle_{\mathrm{VSA}}B_8^{3/2}+C_9\langle O_9^{3/2}\rangle_{\mathrm{VSA}}B_9^{3/2}\Big], \qquad (4)$$

where $\lambda_t=V_{ts}^*V_{td}$ contains the CKM matrix dependence and $\langle O^{3/2}\rangle_{\mathrm{VSA}}$ stands for the VSA matrix element of the corresponding operator. The definitions of the operators can be found for example in Refs. [22,23]. The Wilson coefficients, known to NLO [22–25], are denoted by $C_k\equiv C_k(M_W/\mu)$ and the operators O_k $(k=7,8,9)$ are defined as:

$$\begin{aligned}
O_7^{3/2}&=\left(\bar{s}_\alpha\gamma_\mu^L d_\alpha\right)\left\{\bar{u}_\beta\gamma_\mu^R u_\beta-\bar{d}_\beta\gamma_\mu^R d_\beta\right\}+\left(\bar{s}_\alpha\gamma_\mu^L u_\alpha\right)\left(\bar{u}_\beta\gamma_\mu^R d_\beta\right),\\
O_8^{3/2}&=\left(\bar{s}_\alpha\gamma_\mu^L d_\beta\right)\left\{\bar{u}_\beta\gamma_\mu^R u_\alpha-\bar{d}_\beta\gamma_\mu^R d_\alpha\right\}+\left(\bar{s}_\alpha\gamma_\mu^L u_\beta\right)\left(\bar{u}_\beta\gamma_\mu^R d_\alpha\right),\\
O_9^{3/2}&=\left(\bar{s}_\alpha\gamma_\mu^L d_\alpha\right)\left\{\bar{u}_\beta\gamma_\mu^L u_\beta-\bar{d}_\beta\gamma_\mu^L d_\beta\right\}+\left(\bar{s}_\alpha\gamma_\mu^L u_\alpha\right)\left(\bar{u}_\beta\gamma_\mu^L d_\beta\right),
\end{aligned} \qquad (5)$$

where $\gamma_\mu^R=\gamma_\mu(1+\gamma_5)$ and $\alpha,\beta=1$–3 are colour indices. The definitions of the *B*-parameters of Eqs. (3) and (4) will be given in Section 4.

Two observations are necessary at this point. Firstly, since we are interested in computing the matrix elements $\langle\overline{K}^0|\hat{O}^{\Delta S=2}|K^0\rangle$ and $\langle\pi^+|\hat{O}_k^{3/2}|K^+\rangle$ (with $k=7,8,9$), only the parity-conserving parts of the operators of Eqs. (2) and (5) enter in the calculation. Secondly, on the lattice the above matrix elements are obtained in the standard way by studying the asymptotic behaviour, at large time separations, of hadronic correlation functions of the form $\langle P(y)\hat{O}(0)P(x)\rangle$ (see Eqs. (15) below), with P denoting suitable pseudoscalar densities which we use as meson sources and sinks. The Wick contractions of the quark fields in the correlation functions give rise to diagrams which are both ''eight''-shaped and ''eye''-shaped. The latter, however, cancel in the limit of degenerate up and down quark masses. Since our results are obtained in this limit, complicated subtractions of lower dimensional operators, necessary for the removal of the power divergences of the ''eye''-diagrams, are avoided.

3. Non-perturbative renormalization

The NPM for the evaluation of the renormalization constants of lattice operators consists in imposing suitable renormalization conditions on lattice amputated quark correlation functions [8]. In our case, we compute four-fermion Green functions in the Landau gauge. All external quark lines are at equal momentum p. After amputating and projecting these correlation functions (see Refs. [11] and [19] for details), the renormalization conditions are imposed in the deep Euclidean region at the scale $p^2 = \mu^2$. This renormalization scheme has been recently called the Regularization Independent (RI) scheme [22] (MOM in the early literature) in order to emphasize that the renormalization conditions are independent of the regularization scheme, although they depend on the external states used in the renormalization procedure (and on the gauge). Thus, at fixed cutoff (i.e. fixed β), we compute non-perturbatively the renormalization constants and the renormalized operator $\hat{O}^{\rm RI}(\mu)$ in the RI scheme. In order to obtain the physical amplitudes, which are renormalization group invariant and scheme independent, the renormalized matrix elements must subsequently be combined with the corresponding Wilson coefficients of the effective Hamiltonian. The latter are known in continuum perturbation theory, at the NLO, both in the $\overline{\rm MS}$ scheme [21–24] and in the RI scheme [22]. Although not strictly necessary, since the standard practice consists in giving the B-parameters in the $\overline{\rm MS}$ scheme, we will express our results both in the $\overline{\rm MS}$ and RI schemes. In order to obtain the corresponding operators in the $\overline{\rm MS}$ scheme, $\hat{O}^{\overline{\rm MS}}(\mu)$, the matrix elements of the operators $\hat{O}^{\rm RI}(\mu)$ must be corrected by finite matching coefficients [22]. We stress that, if μ is much larger than $\Lambda_{\rm QCD}$, the NPM of Ref. [11] and the WI method of Ref. [9], used for the computation of the mixing coefficients of the lattice operators, are equivalent, in the chiral limit. This has been shown for two-fermion operators in Ref. [8]; for four-fermion operators it is discussed in detail in Refs. [19,26] (see also Ref. [3]).

In [19], we have determined non-perturbatively the operator mixing for the complete basis of four-fermion operators, with the aid of the discrete symmetries (parity, charge conjugation and switching of flavours), in the spirit of Ref. [4]. The renormalization of the parity-conserving operators, relevant to this work, is conveniently expressed in terms of the following basis of five operators:

$$\begin{aligned} Q_1 &= V\times V + A\times A,\\ Q_2 &= V\times V - A\times A,\\ Q_3 &= S\times S - P\times P,\\ Q_4 &= S\times S + P\times P,\\ Q_5 &= T\times T. \end{aligned} \tag{6}$$

The operators $Q_1,\ldots,Q_5$ form a complete basis on the lattice. In these expressions, $\Gamma\times\Gamma$ (with $\Gamma = V,A,S,P,T$ a generic Dirac matrix) stands for $\frac{1}{2}(\bar{\psi}_1\Gamma\psi_2\bar{\psi}_3\Gamma\psi_4 + \bar{\psi}_1\Gamma\psi_4\bar{\psi}_3\Gamma\psi_2)$, where ψ_i, $i = 1,\ldots,4$ are fermion fields with flavours chosen so as to reproduce the desired operators (see Ref. [19] for details). More specifically, the parity-conserving component of the four-fermion operator $O^{\Delta S=2}$ corresponds to Q_1 in our basis. On the lattice, this operator mixes under renormalization with the other four operators as follows

$$\hat{Q}_1 = Z_{11}\left[Q_1 + \sum_{i=2}^{5} Z_{1i}Q_i\right], \tag{7}$$

where Z_{11} is a multiplicative logarithmically divergent renormalization constant; it depends on the coupling and $a\mu$. The mixing coefficients Z_{1i} (with $i = 2,\ldots,5$) are finite; they only depend on the lattice coupling $g_0^2(a)$.

The renormalization of the parity-conserving parts of the operators $O_7^{3/2}$ and $O_8^{3/2}$ is related to that of the operators Q_2 and Q_3; the correspondence is given by

$$O_7^{3/2} \to Q_2, \quad O_8^{3/2} \to -2Q_3.$$

The renormalized operators are defined as:

$$\hat{Q}_2 = Z_{22}Q_2^s + Z_{23}Q_3^s, \quad \hat{Q}_3 = Z_{32}Q_2^s + Z_{33}Q_3^s, \tag{8}$$

where Z_{ij} (with $i,j = 2,3$) are logarithmically divergent renormalization constants which depend on the coupling and $a\mu$. The above mixing matrix is not peculiar to the lattice regularization, but also occurs in the continuum. The breaking of chiral symmetry by the Wilson action requires the additional subtractions:

$$Q_i^s = Q_i + \sum_{j=1,4,5} Z_{ij}Q_j, \quad i = 2,3. \tag{9}$$

where the Z_{ij}s are finite coefficients which only depend on $g_0^2(a)$.

Finally, the operator $O_9^{3/2}$ corresponds to the operator Q_1 of Eqs. (6). Thus, its renormalization constants, B-parameter, etc. are identical to those of $O^{\Delta S=2}$. The results for all the renormalization constants Z_{ij} (computed with the NPM at several renormalization scales μ at $\beta = 6.0$) can be found in [19].

4. B-parameters

In the $\Delta S = 2$ case, the B-parameter is defined as

$$B_K(\mu) = \frac{\langle \overline{K}^0 | \hat{O}^{\Delta S=2}(\mu) | K^0 \rangle}{\langle \overline{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle_{\rm VSA}} , \qquad (10)$$

where the matrix element in the VSA is given by

$$\begin{aligned} &\langle \overline{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle_{\rm VSA} \\ &\quad = 2\left(1 + \frac{1}{N_c}\right) Z_A^2 |\langle K^0 | A_\mu | 0 \rangle|^2 \\ &\quad = 2\left(1 + \frac{1}{N_c}\right) m_K^2 f_K^2 , \end{aligned} \qquad (11)$$

with Z_A the (finite) renormalization constant of the lattice axial current $A_\mu = \bar{\psi}\gamma_\mu\gamma_5\psi$. Since $\langle \overline{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle_{\rm VSA}$ is given in terms of physical quantities (m_K and f_K), $B(\mu)$ runs with the scale μ exactly like the corresponding renormalized operator $\hat{O}(\mu)$. Another definition of the VSA matrix element has recently been proposed in Ref. [9]. It consists in vacuum-saturating each operator which enters in the subtraction of Eq. (7). In this way, statistical fluctuations are reduced in the ratio of correlation functions used to extract B_K (defined as $R^{\Delta S=2}$ in Eqs. (16) below). As pointed out in Ref. [27], however, the definition used in Ref. [9] spoils the good scaling properties of $B_K(\mu)$. Thus, we insist on retaining the standard definition of Eq. (12), at the price of having larger statistical errors.

For the $\Delta I = 3/2$ transitions, the B-parameters are defined by:

$$\begin{aligned} B_7^{3/2}(\mu) &= \frac{\langle \pi^+ | \hat{O}_7^{3/2}(\mu) | K^+ \rangle}{\langle \pi^+ | \hat{O}_7^{3/2} | K^+ \rangle_{\rm VSA}} , \\ B_8^{3/2}(\mu) &= \frac{\langle \pi^+ | \hat{O}_8^{3/2}(\mu) | K^+ \rangle}{\langle \pi^+ | \hat{O}_8^{3/2} | K^+ \rangle_{\rm VSA}} . \end{aligned} \qquad (12)$$

As discussed in Section 3, $B_9^{3/2}(\mu) = B_K(\mu)$ in the limit of degenerate quark masses. The VSAs for the above matrix elements depend on two different contributions of the form:

$$\begin{aligned} \langle \pi^+ | \hat{O}_7^{3/2} | K^+ \rangle_{\rm VSA} &= \frac{2}{N_c} Z_P^2 \langle \pi^+ | P | 0 \rangle \langle 0 | P | K^+ \rangle \\ &\quad - Z_A^2 \langle \pi^+ | A_\mu | 0 \rangle \langle 0 | A_\mu | K^+ \rangle , \\ \langle \pi^+ | \hat{O}_8^{3/2} | K^+ \rangle_{\rm VSA} &= 2 Z_P^2 \langle \pi^+ | P | 0 \rangle \langle 0 | P | K^+ \rangle \\ &\quad - \frac{Z_A^2}{N_c} \langle \pi^+ | A_\mu | 0 \rangle \langle 0 | A_\mu | K^+ \rangle , \end{aligned} \qquad (13)$$

where Z_P is the renormalization constant of the lattice pseudoscalar density $P = \bar{\psi}\gamma_5\psi$ (renormalized at the same scale μ in the RI scheme). Since we work with degenerate quark masses, we have left the flavour content of the operators A_μ and P unspecified; they are meant to have whatever flavour is required by the hadronic states of their matrix elements (i.e. P_π, P_K and similarly for A_μ). Contrary to the $\Delta S = 2$ case, the leading terms of the above VSA matrix elements are μ-dependent quantities, with an anomalous dimension equal to twice the anomalous dimension of the pseudoscalar density P. Thus the B-parameters do not scale in μ like the matrix elements of the corresponding operators. The last terms on the r.h.s. of Eqs. (14) vanish in the chiral limit. Consequently, since we are ultimately interested in passing from the $\langle \pi^+ | \hat{O}^{3/2} | K^+ \rangle$ matrix elements to the $\langle \pi\pi | \hat{O}^{3/2} | K \rangle$ ones using soft pion theorems, following Ref. [16], we have dropped the last terms on the r.h.s. of Eqs. (14). In order to extract the B-parameters, we need to compute the following two- and three-point correlation functions:

$$\begin{aligned} G_P(t_x, \boldsymbol{p}) &= \sum_{\boldsymbol{x}} \langle P(x) P(0) \rangle e^{-\boldsymbol{p}\cdot\boldsymbol{x}} , \\ G_A(t_x, \boldsymbol{p}) &= \sum_{\boldsymbol{x}} \langle A_0(x) P(0) \rangle e^{-\boldsymbol{p}\cdot\boldsymbol{x}} , \\ G_{\hat{O}}(t_x, t_y; \boldsymbol{p}, \boldsymbol{q}) &= \sum_{\boldsymbol{x},\boldsymbol{y}} \langle P(y) \hat{O}(0) P(x) \rangle e^{-\boldsymbol{p}\cdot\boldsymbol{y}} e^{\boldsymbol{q}\cdot\boldsymbol{x}} , \end{aligned} \qquad (14)$$

where $x \equiv (\boldsymbol{x}, t_x), y \equiv (\boldsymbol{y}, t_y)$ and $\hat{O}$ stands for any four-fermion operator of interest. As stated above, all

correlation functions have been evaluated with degenerate quark masses and therefore only the ''eight-diagrams'' contribute to $G_{\hat{O}}$. By forming suitable ratios of the above correlations, and looking at their asymptotic behaviour at large time separations, we isolate the desired operator matrix elements. In particular the ratios:

$$R^{\Delta S=2} = \frac{1}{Z_A^2}\frac{G_{\hat{O}^{\Delta S=2}}}{G_P G_P} \to \frac{\langle \bar{K}^0(\boldsymbol{p})|\hat{O}^{\Delta S=2}|K^0(\boldsymbol{p})\rangle}{Z_A^2|\langle 0|P|K^0\rangle|^2},$$

$$R_7^{3/2} = -\frac{N_c}{2Z_P^2}\frac{G_{\hat{O}_7^{3/2}}}{G_P G_P} \to \frac{N_c}{2}\frac{\langle \pi^+|\hat{O}_7^{3/2}|K^+\rangle}{Z_P^2\langle \pi^+|P|0\rangle\langle 0|P|K^+\rangle},$$

$$R_8^{3/2} = -\frac{1}{2Z_P^2}\frac{G_{\hat{O}_8^{3/2}}}{G_P G_P} \to \frac{1}{2}\frac{\langle \pi^+|\hat{O}_8^{3/2}|K^+\rangle}{Z_P^2\langle \pi^+|P|0\rangle\langle 0|P|K^+\rangle}, \quad (15)$$

give, up to computable factors, the B-parameters of interest. For comparison, we have obtained results with the operator renormalized not only with the NPM, but also with SPT and BPT.

5. Numerical results

Our simulation has been performed at $\beta = 6.0$ with the tree-level Clover action in the quenched approximation. Quark masses have been fixed at $\kappa = 0.1440, 0.1432$ and 0.1425. The renormalization constants have been obtained from quark correlation functions, in the Landau gauge, on a $16^3 \times 32$ lattice, with 100 configurations. The hadronic matrix elements have been computed on an $18^3 \times 64$ lattice with 460 configurations. Details on the choice of time intervals, spatial momenta and related technicalities are to be found in Ref. [12]. Statistical errors have been estimated with the jacknife method, by decimating 46 configurations at a time. We have neglected the statistical errors of the renormalization constants, quoting only those of the hadronic matrix elements. In the above ratios, we also need the (finite) axial-current renormalization constant Z_A and the $a\mu$-dependent renormalization constant Z_P of the pseudoscalar density. Depending on the method of renormalization of the four-fermion operator (NPM, SPT or BPT), we have used the corresponding estimate of Z_A and Z_P, obtained with the same method of calculation. Although Z_A should not depend [2] on $a\mu$, slight variations of its NPM estimate, arising from systematic effects, partially cancel analogous variations of $R^{\Delta S=2}$, giving much more stable results. The NPM estimates for Z_P and Z_A used in the present work are those of Ref. [29].

In order to extract the B-parameters from the ratios of Eqs. (16), we follow the procedure of Ref. [12], fitting them with the function

$$R = \alpha + \beta X + \gamma Y, \quad (16)$$

where

$$X = \frac{8}{3}\frac{G_A G_A^\dagger}{G_P G_P} \to \frac{8}{3}\frac{f_K^2 m_K^2}{Z_A^2|\langle 0|P|K^0\rangle|^2},$$

$$Y = \frac{(p\cdot q)}{m_K^2}X \quad (17)$$

(the large time asymptotic behaviour of X is also shown above). The results of the fits are collected in Table 1, at eight different values of the renormalization scale $\mu^2 a^2$. We also show results obtained with the renormalization constants evaluated in SPT and in BPT. In the latter case, we use the recipe $\alpha_s^{\rm boost} = \alpha_s/(\frac{1}{3}\,{\rm Tr}\langle U_\Box\rangle)$, where $\langle U_\Box\rangle$ stands for the average plaquette. At $\beta = 6.0$, we have used $\alpha_s^{\rm boost} = 1.68/4\pi$.

We first examine the results for $R^{\Delta S=2}$, from which B_K can be extracted. As in [11–13], it can be seen that α and β (which are lattice artifacts) are compatible with zero within at most 2σ, unlike their SPT and BPT values. Thus (within our statistical accuracy and up to terms of $\mathcal{O}(g^2 a)$), the correct chiral behaviour of the matrix elements is restored using the NPM. We point out the stability of γ as a function of μ. Assuming α and β to be zero, the B_K-parameter is then given by:

$$B_K = \gamma. \quad (18)$$

In our original analysis [11,12], we have erroneously considered the mixing of the $\Delta S = 2$ opera-

[2] For the tree-level Clover action, the best available non-perturbative estimate for Z_A is obtained from lattice axial WI's; at $\beta = 6.0$ this is $Z_A = 1.11(2)$ [28].

tor with only the three other operators which appear in one-loop perturbation theory [2]. It has been stressed in [9,13,14,19] that, non-perturbatively, $O^{\Delta S=2}$ also mixes with a fourth operator. The results of Ref. [13] and the present work take into account the complete non-perturbative mixing.

We now turn to $R_7^{3/2}, R_8^{3/2}$, from which $B_7^{3/2}$ and $B_8^{3/2}$ can be extracted. Note the stability of α as a function of $a\mu$ and the compatibility of β with zero. In this case, for both operators, the B-parameters are simply given by

$$B^{3/2} = \alpha. \tag{19}$$

Comparing the NPM results to those obtained in SPT and BPT, we find agreement between the NPM and SPT values of $B_7^{3/2}$, whereas its BPT value is incompatible with the other two. The $B_8^{3/2}$ result depends on the method used for its renormalization.

Table 1
Values of the fit parameters for $R^{\Delta S=2}, R_7^{3/2}$ and $R_8^{3/2}$, from the NPM (at several renormalization scales), SPT and BPT

Operator	$\mu^2 a^2$	α	β	γ
$R^{\Delta S=2}$	0.31	0.027 ± 0.014	0.17 ± 0.16	0.68 ± 0.12
	0.62	-0.018 ± 0.014	0.28 ± 0.17	0.67 ± 0.12
	0.96	-0.014 ± 0.014	0.27 ± 0.16	0.66 ± 0.11
	1.27	-0.010 ± 0.013	0.26 ± 0.16	0.66 ± 0.11
	1.39	-0.004 ± 0.013	0.23 ± 0.16	0.66 ± 0.11
	1.85	-0.005 ± 0.013	0.25 ± 0.16	0.66 ± 0.12
	2.46	0.002 ± 0.013	0.25 ± 0.16	0.67 ± 0.12
	4.01	0.012 ± 0.012	0.25 ± 0.16	0.68 ± 0.12
	SPT	-0.069 ± 0.013	0.17 ± 0.16	0.65 ± 0.12
	BPT	-0.058 ± 0.013	0.18 ± 0.16	0.66 ± 0.12
$R_7^{3/2}$	0.31	1.70 ± 0.16	-6.4 ± 1.3	6.3 ± 0.7
	0.62	0.81 ± 0.07	-0.8 ± 0.6	3.5 ± 0.4
	0.96	0.69 ± 0.04	-0.1 ± 0.4	2.6 ± 0.3
	1.27	0.70 ± 0.03	0.1 ± 0.4	2.3 ± 0.3
	1.39	0.70 ± 0.03	0.04 ± 0.34	2.2 ± 0.3
	1.85	0.69 ± 0.03	0.15 ± 0.28	1.94 ± 0.23
	2.47	0.69 ± 0.02	0.22 ± 0.25	1.77 ± 0.21
	4.01	0.73 ± 0.02	0.31 ± 0.22	1.62 ± 0.20
	SPT	0.68 ± 0.02	0.45 ± 0.17	1.14 ± 0.14
	BPT	0.48 ± 0.02	0.41 ± 0.22	1.58 ± 0.17
$R_8^{3/2}$	0.31	1.20 ± 0.06	-1.0 ± 0.4	0.66 ± 0.20
	0.62	1.08 ± 0.04	-0.5 ± 0.3	0.56 ± 0.15
	0.96	1.04 ± 0.03	-0.3 ± 0.3	0.53 ± 0.14
	1.27	1.00 ± 0.03	-0.06 ± 0.25	0.52 ± 0.13
	1.39	1.00 ± 0.03	-0.05 ± 0.25	0.52 ± 0.13
	1.85	0.99 ± 0.03	0.04 ± 0.24	0.51 ± 0.13
	2.46	0.99 ± 0.02	0.09 ± 0.24	0.51 ± 0.13
	4.01	0.98 ± 0.02	0.15 ± 0.23	0.51 ± 0.13
	SPT	0.81 ± 0.02	0.36 ± 0.17	0.42 ± 0.09
	BPT	0.75 ± 0.02	0.29 ± 0.16	0.39 ± 0.09

6. Physics results and conclusions

The B-parameters which can be extracted from the results given in Table 1 have been obtained in the RI scheme, since the operators have been renormalized with the NPM.

As previously stated, it is customary to express all results in the $\overline{\text{MS}}$ scheme (with NDR dimensional regularization). The finite matching between the RI and $\overline{\text{MS}}$-NDR renormalization schemes can be done in continuum perturbation theory, by computing the operator matrix element in the same gauge and on the same external quark states as those used for the non-perturbative calculation of the lattice renormalization constants. At NLO this matching has been obtained in [22]. We have used it in order to convert our NPM results from the RI to the $\overline{\text{MS}}$-NDR scheme. The SPT and BPT cases are less straightforward: we have started from the renormalization constants of Ref. [30], which relate the lattice operators to those renormalized in the $\overline{\text{MS}}$-DRED scheme. To these constants we have subtracted the contributions due to the use of the DRED scheme and added those corresponding to the RI one. In this way we have obtained, for both the SPT and BPT cases, the renormalization constants in the RI scheme, from which the results of Table 1 are computed. Alternatively, we have also calculated and added the necessary $\overline{\text{MS}}$-NDR contributions in order to obtain the SPT and BPT results in this scheme. The latter calculation has also been performed in [14] for the Wilson action. At NLO [22], the result for B_K is

$$B_K^{\overline{\text{MS}}}(\mu) = \left(1 - \frac{\alpha_s(\mu)}{4\pi} \Delta r_+^{\overline{\text{MS}}}\right) B_K^{\text{RI}}(\mu), \tag{20}$$

where

$$\Delta r_+^{\overline{\text{MS}}} = 14/3 - 8\ln 2. \tag{21}$$

The matching relation for the renormalized operators $\hat{O}_i^{3/2}$ ($i=7,8$) at NLO is also given in [22]:

$$\left(\hat{O}_i^{3/2}\right)_{\overline{\rm MS}} = \left(\delta_{ij} - \frac{\alpha_s(\mu)}{4\pi}\Delta r_{ij}^{\overline{\rm MS}}\right)\left(\hat{O}_j^{3/2}\right)_{\rm RI} \qquad (22)$$

where

$$\Delta r_{ij}^{\overline{\rm MS}} = \begin{pmatrix} \frac{2}{3}+\frac{2}{3}\ln 2 & -2-2\ln 2 \\ 2-2\ln 2 & -\frac{34}{3}+\frac{2}{3}\ln 2 \end{pmatrix}. \qquad (23)$$

Hence, for the B-parameters $B_7^{3/2}$ and $B_8^{3/2}$ we have:

$$\begin{aligned}\begin{pmatrix} B_7^{3/2} \\ B_8^{3/2} \end{pmatrix}_{\overline{\rm MS}} &= \frac{1}{\left(1+\frac{\alpha_s(\mu)}{4\pi}\Delta r_P^{\overline{\rm MS}}\right)^2} \\ &\times \begin{pmatrix} 1-\frac{\alpha_s(\mu)}{4\pi}\Delta r_{77}^{\overline{\rm MS}} & -N_c\frac{\alpha_s(\mu)}{4\pi}\Delta r_{78}^{\overline{\rm MS}} \\ -\frac{1}{N_c}\frac{\alpha_s(\mu)}{4\pi}\Delta r_{87}^{\overline{\rm MS}} & 1-\frac{\alpha_s(\mu)}{4\pi}\Delta r_{88}^{\overline{\rm MS}} \end{pmatrix} \\ &\times \begin{pmatrix} B_7^{3/2} \\ B_8^{3/2} \end{pmatrix}_{\rm RI} \qquad (24)\end{aligned}$$

with

$$\Delta r_P^{\overline{\rm MS}} = 16/3. \qquad (25)$$

The uncertainty due to the choice of $\alpha_s(\mu)$ is about ± 0.03. We find a further uncertainty of ± 0.03 when varying the number of active flavours from 0 to 4.

In order to compare our result for B_K with other theoretical predictions of the same quantity, it is useful to convert it to the RGI quantity, $\hat{B}_K$, by multiplying it by the Wilson coefficient. At NLO, $\hat{B}_K$ is given by

$$\begin{aligned}\hat{B}_K &= \alpha_s(\mu)^{-\gamma^{(0)}/2\beta_0} \\ &\times \left[1-\frac{\alpha_s(\mu)}{4\pi}\left(\frac{\gamma^{(1)}\beta_0-\gamma^{(0)}\beta_1}{2\beta_0^2}\right)\right] B_K^{\overline{\rm MS}}(\mu), \qquad (26)\end{aligned}$$

where $\beta_{0,1}$ and $\gamma^{(0,1)}$ are the leading and next-to-leading coefficients of the β-function and anomalous dimension. $\beta_{0,1}$ and $\gamma^{(0)}$ are universal whereas $\gamma^{(1)}$ depends on the regularization and renormalization schemes. The explicit expressions of these quantities can be found for example in [21]. This estimate of $\hat{B}_K$ is also regularization-scheme independent, up to next-to-NLO order terms.

Table 2
B-parameters for $\Delta S=2$ and $\Delta I=3/2$ operators at the renormalization scale $\mu = a^{-1} \simeq 2$ GeV. All results are in the $\overline{\rm MS}$ scheme

Quantity	Method	Result	Ref.
B_K	NPM	0.66(11)	this work
	BPT	0.65(11)	this work
	BPT $q^* = 1/a$	0.74(4)	[14]
$\hat{B}_K$	NPM	0.93(16)	this work
	BPT	0.92(16)	this work
$B_7^{3/2}$	NPM	0.72(5)	this work
	BPT	0.58(2)	this work
	BPT $q^* = 1/a$	0.58(2)	[14]
	BPT $q^* = \pi/a$	0.65(2)	[14]
$B_8^{3/2}$	NPM	1.03(3)	this work
	BPT	0.83(2)	this work
	BPT $q^* = 1/a$	0.81(3)	[14]
	BPT $q^* = \pi/a$	0.84(3)	[14]

In Table 2 we collect all our results for the B-parameters, obtained by computing their renormalization with the NPM and in BPT, at the reference scale $\mu \simeq 2$ GeV in the $\overline{\rm MS}$ scheme. For B_K we also give the RGI value $\hat{B}_K$. Moreover, we compare our results to the most recent ones obtained with the Wilson action at the same coupling and with operator renormalizations carried out perturbatively in BPT [14].

Two points concerning this comparison deserve some attention here. The first is that in Ref. [14] the results were obtained in 1-loop BPT by renormalizing the lattice operators directly in the $\overline{\rm MS}$-NDR scheme, rather than doing the renormalization in the RI scheme and then matching to $\overline{\rm MS}$-NDR with Eq. (25), as we have to do with the NPM. In perturbation theory, the two procedures differ by $\mathcal{O}(\alpha_s^2)$ terms, which introduce a systematic difference of about 12%–15% in the results for $B_7^{3/2}$ and $B_8^{3/2}$. For this reason, in order to make the comparison meaningful, our BPT estimates in the $\overline{\rm MS}$-NDR scheme, given in

Table 2, were obtained using the same procedure as in [14]. The second point is that in Ref. [14], a more complicated version of BPT than in this work has been implemented, which involves tadpole resummations and a choice of ''optimal'' scale q^* for the boosted coupling. Since the recipe for q^* is not unique, results with several choices of q^* have been listed in [14], and can be compared with our results. Finally, we point out that a direct comparison of our B_K value to those of [9] is not possible, because the latter have been obtained at different lattice couplings.

Our results from NPM and BPT for B_K and $\hat{B}_K$ are in perfect agreement. With a larger statistical error, our B_K value also agrees with those of Refs. [14,31]. We find, instead, a discrepancy between our NPM and BPT estimates of $B_7^{3/2}$ and $B_8^{3/2}$. Our values obtained with BPT are fully compatible to those of [14] (at least for one q^* value), where the Wilson action was used. The NPM estimate, instead, is in disagreement with any value obtained in BPT (either with the Wilson or the Clover action and for several boosting variants). This indicates that the difference between our NPM estimate and that of Ref. [14] is due to the NPM used in the former result, rather than the implementation of different actions (Clover and Wilson respectively). The increase in the value of $B_8^{3/2}$, obtained with the NPM, is of great phenomenological interest, since it may induce a significant decrease of the ratio ϵ'/ϵ.

In conclusion, the non-perturbative renormalization of the four-fermion operators, either with the NPM or with WI's, strongly improves the reliability of lattice computations of B-parameters with Wilson-like fermions. Perturbative mixing of lattice operators, carried out at lowest order, fails to reproduce the expected chiral behaviour of the matrix elements (at present day couplings). A good chiral behaviour of $O^{\Delta S=2}$ is restored when the operators are renormalized non-perturbatively. Moreover, $B_7^{3/2}$ and $B_8^{3/2}$ obtained with the NPM disagree by as much as 20% from the ones obtained in BPT.

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