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## The $t \rightarrow WZb$ decay in the Standard Model: a critical reanalysis

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#### Abstract

We compute the  $t \to WZb$  decay rate, in the Standard Model, at the leading order in perturbation theory, with special attention to the effects of the finite widths of the W and Z bosons. These effects are extremely important, since the  $t \to WZb$  decay occurs near its kinematical threshold. They increase the value of the decay rate by orders of magnitude near threshold or allow it below the nominal threshold. We discuss a procedure to take into account the finite-width effects and compare the results with previous studies of this decay. Within the Standard Model, for a top quark mass in the range between 170 and 180 GeV, we find  $BR(t \to WZb) \simeq 2 \times 10^{-6}$ , which makes the observation at the LHC very difficult if at all possible. © 2001 Elsevier Science B.V. All rights reserved.

#### 1. Introduction

The  $t \rightarrow WZb$  process is a particularly interesting rare decay mode. In the Standard Model, all rare top decays are extremely suppressed with respect to the largely dominant  $t \rightarrow Wb$  decay. As possibly it could be observable at the LHC, the  $t \rightarrow WZb$ decay has already attracted considerable attention in the literature [1–3].

The most peculiar feature of the  $t \rightarrow WZb$  decay is the fact that the process occurs near the kinematical threshold,  $m_t \simeq m_W + m_Z + m_b$ . As observed in Ref. [2], it is then crucial, in the theoretical evaluation of the decay rate, to take into account the finite-width effects for the W and Z bosons. For a top quark mass around approximately 175 GeV, these effects increase the value of the decay rate by some orders of magnitude.

Even with the sizeable enhancement of the rate induced by finite-width effects, the branching ratio for  $t \rightarrow WZb$  is predicted to be of the order of  $10^{-6}$ , which could be too small for the decay to be observed even at the LHC, where  $10^7-10^8$  top quark pairs are expected to be produced per year. Therefore, the observation of this decay mode at the LHC could be a signal of physics beyond the Standard Model (see, for example, Refs. [4,5]).

In this paper we first discuss the results for the  $t \rightarrow WZb$  decay rate computed at tree-level, in the Standard Model, by neglecting the effects of the finite widths of the W and Z bosons, i.e., by treating these particles as stable particles in the final state. This quantity has also been computed in Refs. [1–3] but the results of these studies are in disagreement. Our results, in the limit of vanishing widths, will be presented in Section 2. They agree with those obtained

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in Ref. [3], but present numerical differences with respect to those of Ref. [1] and, near threshold, of Ref. [2].

In Section 3, we compute the  $t \rightarrow WZb$  decay rate by taking into account, as in Ref. [2], the finitewidth effects of the W and Z bosons. We consider two different approaches. The first approach, which we will refer to as the *convolution method*, has been discussed, in a different context, in Ref. [6]. It is based on a convolution of the  $t \rightarrow WZb$  decay rate with two Breit–Wigner-like distributions for the invariant masses of the W and Z bosons. These distributions correspond to the imaginary part of the gauge boson propagators, and are centered around the physical values of the W and Z masses with spread controlled by the physical gauge boson widths.

The second approach has been followed in Ref. [2]. It consists in considering the decay rate for the process  $t \rightarrow W^*Z^*b$  followed by the decays of the virtual W and Z bosons, e.g.,  $W^* \rightarrow \mu \nu_{\mu}$  and  $Z^* \rightarrow e^+e^-$ . The  $t \rightarrow WZb$  decay rate is then obtained by dividing the rate computed for the full decay chain by the product of the  $W \rightarrow \mu \nu_{\mu}$  and  $Z \rightarrow e^+e^-$  branching ratios. We will refer to this second approach as the *decay-chain method*. Incidentally, we note that one Feynman diagram entering at tree-level has not been included in the calculation of Ref. [2].

A difficulty inherent to the decay-chain method is that other initial processes, besides the  $t \to WZb$ decay of interest, may contribute to the decay chain. For instance, when an electron pair is considered in the final state, a virtual photon, instead of the Z boson, may be produced in the intermediate states. Therefore, in this case, the direct connection between the full  $t \rightarrow b \mu \nu_{\mu} e^+ e^-$  process (total number of events) and the  $t \rightarrow WZb$  decay ("signal") is lost. In order to suppress the contribution of the virtual photon, a kinematical cut on the invariant mass of the electron pair (requiring  $m_{ee} \ge 0.8 m_Z$ ) has been introduced in Ref. [2]. This cut is indeed effective and, in the selected region of the phase space, the  $t \rightarrow b \mu v_{\mu} e^+ e^$ process mostly proceeds through the initial  $t \rightarrow WZb$ decay. However, with this requirement, the definition itself of the  $t \rightarrow WZb$  decay rate becomes dependent on the specific choice of the cut. The purpose of this paper is to show that, by using the convolution method, a convenient definition of the  $t \rightarrow WZb$  decay rate can be provided, which is independent of any kinematical

cut. In this way, we find results for  $\Gamma(t \to WZb)$  which differ by approximately a factor 3 with respect to those obtained following the procedure of Ref. [2]. Of course, the full calculation of  $t \to b\mu\nu_{\mu}e^+e^-$ , and the analysis of the kinematical cuts, can be relevant for the evaluation of the measurable signal plus background rate.

The calculation of the QCD radiative corrections goes beyond the scope of the present Letter. However, their effect could be important and give rise to a further reduction of the rate due to Sudakov suppression factors that tend to become dominant near the end of phase space.

# 2. The $t \rightarrow WZb$ decay rate in the limit of stable *W* and *Z* bosons

Within the Standard Model, three Feynman diagrams contribute to the  $t \rightarrow WZb$  decay at the leading order in perturbation theory. These diagrams are shown in Fig. 1 and represent the amplitudes with the final Z boson radiated either by the initial top, or by the final b or W respectively.

We have computed the decay rate for this process by considering first the final W and Z bosons as stable particles, i.e., neglecting their finite widths. In Table 1 we collect the values of the various quantities which have been used to obtain all the results presented in this Letter. The integration over the final phase space of the three massive particles has been performed numerically. For the values of masses considered in this Letter, the kinematical threshold for the process is at  $m_t = 176$  GeV. Note that the exact threshold value depends on  $m_b$ . At the parton level,  $m_b$  is in principle scale dependent. A low scale appears justified here because of both the very limited phase space allowed to a virtual b and the value of lightest B-meson mass.



Fig. 1. Feynman diagrams contributing at tree-level to the  $t \rightarrow WZb$  decay.

values of the numerical constants used in this Letter				
m <sub>b</sub>	$m_W$	mZ	$\sin^2 \theta_W$	$G_F$
4.5 GeV	80.3 GeV	91.2 GeV	0.23	$1.166 \times 10^{-5} \text{ GeV}^{-2}$
$\Gamma_W$	$\Gamma_Z$	$BR(W \to \mu \nu_{\mu})$	$BR(Z \to e^+ e^-)$	$BR(Z \to \nu \bar{\nu})$
2.06 GeV	2.49 GeV	0.102	0.03367	0.06667

Table 1 Values of the numerical constants used in this Letter

In the limit of neglecting the finite widths of the W and Z bosons, the decay is forbidden below the threshold. For larger values of the top quark mass, we obtain the values of the branching ratio presented in Table 2. <sup>1</sup> The uncertainties on these results, coming from the numerical integrations, are estimated to be smaller than the last digit shown in the table. For values of the top quark mass around 180 GeV, we find that the branching ratio is extremely small, of the order of  $10^{-8}$ .

The results obtained in this section agree with those of Ref. [3]. We also agree with the analytical expression of the tree-level Feynman amplitude squared published in that paper. On the other hand, we disagree with the numerical results of Ref. [1] (we note that there is an apparent error in the integration limits over the phase space of the three massive particles (Eq. (40) of [1])).

We also found discrepancies, of the order of 50%, with respect to the results obtained in Ref. [2] in what they call the *narrow-width approximation*. This procedure consists in deriving the values of the  $t \rightarrow WZb$ decay rate starting from the calculation of  $\Gamma(t \rightarrow b\mu\nu e^+e^-)$  and constraining the invariant masses of the virtual *W* and *Z* bosons to their physical values. In turn, these constraints are achieved by introducing *ad hoc* delta functions into the phase space integrals. We find that, in the stable particles limit, our results differ from those of Ref. [2] by a factor 2 for  $m_{top} = 177$ GeV and by approximately 10% for  $m_{top} = 185$  GeV.

$$BR(t \to WZb) \equiv \frac{\Gamma(t \to WZb)}{\Gamma(t \to Wb)}.$$

# 3. The $t \rightarrow WZb$ decay rate including the finite-width effects

In this section we discuss the calculation of the  $t \rightarrow WZb$  decay rate by taking into account the large finite-width effects of the *W* and *Z* bosons. We present in detail the two approaches of the convolution and decay-chain method, and critically compare the corresponding estimates for the decay rate.

#### 3.1. The convolution method

Since the *W* and *Z* bosons are unstable particles with finite widths, their production, in a physical process, can be approximately described as the production of real stable particles with invariant masses distributed according to a given distribution function. The central value and the width of such a distribution are controlled by the physical mass and width of the unstable particle. From this point of view, for instance, the  $t \rightarrow WZb$  decay, if occurs for values of the top quark mass smaller than the kinematical threshold, proceeds through the production of *W* and *Z* bosons with invariant masses close, but smaller than, their physical values.

These observations suggest a very convenient way to take into account the finite-width effects in the  $t \rightarrow WZb$  decay and, in general, in any process with unstable particles produced in the final state. One computes the decay rate of the  $t \rightarrow WZb$  channel as a function of generic values of the W and Z invariant masses. Let us denote this quantity as  $\Gamma(k_W^2, k_Z^2)$ , where  $k_W^2$  and  $k_Z^2$  are the virtualities of the W and Z bosons respectively. The  $t \rightarrow WZb$  decay rate is then obtained by performing a convolution of  $\Gamma(k_W^2, k_Z^2)$ 

<sup>&</sup>lt;sup>1</sup> Throughout this Letter, we approximate the total decay width of the top quark with the partial rate  $\Gamma(t \to Wb)$ , computed at the corresponding values of the top quark mass. Therefore, we define for instance the  $t \to WZb$  branching ratio as:

Table 2

Values of the  $t \to WZb$  branching ratio, as a function of the top quark mass, obtained in the limit of vanishing Z and W widths (stable particles) and with the convolution method. In the last column, the estimate obtained with the decay-chain method from the  $t \to b\mu\nu_{\mu}\nu\bar{\nu}$  decay is also shown for comparison

<i>m</i> <sub>top</sub>	$BR(t \to WZb)$ Stable particles	$BR(t \rightarrow WZb)$ Convolution method	$\frac{BR(t \to b\mu\nu_{\mu}\nu\bar{\nu})}{BR(W \to \mu\nu_{\mu})BR(Z \to \nu\bar{\nu})}$
150	_	$0.65(3) \times 10^{-6}$	$0.79(1) \times 10^{-6}$
155	_	$0.90(5) \times 10^{-6}$	$0.87(1) \times 10^{-6}$
160	_	$0.99(5) \times 10^{-6}$	$1.06(3) \times 10^{-6}$
165	_	$1.40(7) \times 10^{-6}$	$1.37(3) \times 10^{-6}$
170	-	$1.81(9) \times 10^{-6}$	$1.53(4) \times 10^{-6}$
175	-	$2.4(1) \times 10^{-6}$	$1.96(5) \times 10^{-6}$
180	$0.07 \times 10^{-6}$	$3.1(2) \times 10^{-6}$	$2.76(8) \times 10^{-6}$
185	$0.63 \times 10^{-6}$	$4.7(2) \times 10^{-6}$	$4.0(2) \times 10^{-6}$
190	$2.20\times10^{-6}$	$7.4(4) \times 10^{-6}$	$6.0(3) \times 10^{-6}$
195	$5.37 \times 10^{-6}$	$11.3(6) \times 10^{-6}$	$9.5(5) \times 10^{-6}$
200	$10.7 \times 10^{-6}$	$17.7(9) \times 10^{-6}$	$18(2) \times 10^{-6}$

with the two invariant-mass distribution functions

$$\Gamma(t \to WZb) = \int_{0}^{(m_t - m_b)^2} dk_Z^2 \int_{0}^{(m_t - m_b - \sqrt{k_Z^2})^2} dk_W^2$$
$$\times \Gamma(k_W^2, k_Z^2)$$
$$\times \rho(k_Z^2, M_Z, \Gamma_Z) \rho(k_W^2, M_W, \Gamma_W). \tag{1}$$

The distribution  $\rho(k^2, M, \Gamma)$  is related to the imaginary part of the gauge boson propagator:

$$\rho(k^2, M, \Gamma) = -\frac{1}{\pi} \operatorname{Im} \left( \frac{1}{k^2 - M^2 + iM\Gamma(k^2)} \right)$$
$$= \frac{1}{\pi} \frac{M\Gamma(k^2)}{(k^2 - M^2)^2 + M^2\Gamma(k^2)^2}, \quad (2)$$

where *M* is the mass of the particle and  $M\Gamma(k^2)$  the imaginary part of the vacuum polarization. In the calculation of  $\Gamma(k^2)$  one may neglect, as a first approximation, the masses of the particles flowing in the loops. In this limit, by simple dimensional

analysis, one gets:

$$M\Gamma(k^2) = k^2 \frac{\Gamma}{M},\tag{3}$$

where  $\Gamma \equiv \Gamma(M^2)$  is the particle width.<sup>2</sup>

In the limit of vanishing width, the distribution function (2) reduces to the delta function  $\delta(k^2 - M^2)$ , and the final particles are constrained on their mass shell. In this limit, Eqs. (1) and (2) are just a consequence of the optical theorem and the Cutkosky rule to

$$\sum_{\lambda=1,2} \epsilon_{\lambda}^{\mu}(k) \epsilon_{\lambda}^{\nu*}(k) = -g^{\mu\nu} + \frac{k^{\mu}k^{\nu}}{k^2}.$$

In turn, this prescription for the sum over the polarization states is necessary to guarantee the positivity of the "off-shell" decay rate  $\Gamma(k_W^2, k_Z^2)$ . We also note that in the calculation of  $\Gamma(k_W^2, k_Z^2)$ , since the invariant mass of the *W*-boson is allowed to vanish in Eq. (1), we have modified the tree-level expression of the *W*-propagator as indicated in Eq. (5).

<sup>&</sup>lt;sup>2</sup> Note that the full  $k^2$  dependence of  $\Gamma(k^2)$  must be taken also into account in the numerator of Eq. (2). This dependence cancels out the singularity at  $k^2 = 0$  appearing in the propagators and also in the sum over the polarization vectors of a particle with invariant mass  $k^2$ :



Fig. 2. The  $t \rightarrow WZb$  branching ratio, as a function of the top quark mass, as obtained in the stable particle limit and by taking into account the *W* and *Z* finite width effects with the convolution method.

extract the imaginary part of the physical amplitude. In general, the procedure outlined above, which we refer to as the *convolution method*, represents the simplest way to take into account the finite-width effects in the evaluation of the decay rate, and it has been already considered in the literature in different contexts (see, for example, Refs. [6,7]).

The results obtained for the  $t \rightarrow WZb$  branching ratio, by using the convolution method, are presented in Table 2. These values are also shown in Fig. 2, together with the corresponding values obtained in the stable-particles approximation, i.e., by neglecting the finite-width effects. We see that, for a top quark mass around the kinematical threshold of 176 GeV, the effects of the finite widths increase the branching ratio by orders of magnitude, and allow the occurrence of the decay even below the threshold. However, despite the large enhancement induced by the widths, for the actual value of the top quark mass, the branching ratio is found to be of the order of  $10^{-6}$ , which is probably too small for the decay to be visible at the Tevatron Run II or even at the LHC.

The finite-width effects in the  $t \rightarrow WZb$  decay rate have also been studied in Ref. [2]. By comparing our results with those obtained following the procedure of Ref. [2], we find that the values of the decay rate are larger by approximately a factor 3. The reason of such a discrepancy will be discussed in detail in the next section.

### 3.2. The decay-chain method

The procedure considered in Ref. [2] to take into account the finite-width effects of the gauge bosons in the  $t \rightarrow WZb$  decay is based on the study of a process which includes the decays of the unstable particles. For illustrative purposes, it is convenient to consider first the decay chain:

$$t \to b W^* Z^* \Longrightarrow W^* \to \mu \nu_\mu, \quad Z^* \to \nu \bar{\nu}$$
 (4)

which differs, from the one studied in Ref. [2], for the presence of a  $\nu \bar{\nu}$  pair, rather than  $e^+e^-$ , in the final state.

In the calculation of the total rate  $\Gamma(t \rightarrow b\mu\nu_{\mu}\nu\bar{\nu})$ , the finite-width effects of the intermediate W and Zbosons are then taken into account by correcting the tree-level expression of the gauge boson propagators. In the unitary gauge, by neglecting the  $k^2$ -dependence of the imaginary part of the vacuum polarization, the modified propagator can be written in the form [8,9]:

$$\frac{i}{k^2 - (m - i\Gamma/2)^2} \left( -g^{\mu\nu} + \frac{k^{\mu}k^{\nu}}{(m - i\Gamma/2)^2} \right), \quad (5)$$

where the correction to the longitudinal part of the propagator is necessary to respect gauge-invariance. In order to preserve unitarity at high energies, the absorbitive part of the triple gauge boson vertex should be also included in the amplitude [10,11]. However, in the case of interest, this correction is numerically negligible and for simplicity it has been omitted closely following the calculation of Ref. [2].

The total  $t \rightarrow b \mu \nu_{\mu} \nu \bar{\nu}$  rate is then used to compute the ratio:

$$\frac{\Gamma(t \to b\mu\nu_{\mu}\nu\bar{\nu})}{BR(W \to \mu\nu_{\mu})BR(Z \to \nu\bar{\nu})}$$
(6)

which is assumed to be an estimate of the  $t \rightarrow WZb$  decay rate.

A crucial requirement, for the above procedure to be consistent, is that the full decay chain, which is considered to estimate  $\Gamma(t \rightarrow WZb)$ , can, at least dominantly, only proceed through the initial  $t \rightarrow$ WZb decay. Otherwise, any direct connection with this process is obviously lost. This requirement is only partially satisfied in the case of the  $t \rightarrow b\mu\nu_{\mu}\nu\bar{\nu}$  decay. Three Feynman diagrams, describing at treelevel the  $t \to b \mu v_{\mu} v \bar{v}$  decay (with  $v \neq v_{\mu}$ ), are those shown in Fig. 1, modified to account for the W and Z bosons decays into the  $\mu v_{\mu}$  and  $\nu \bar{\nu}$  pairs, respectively. These diagrams describe the processes which proceed through the initial  $t \rightarrow WZb$  decay. Besides these graphs, however, other three Feynman diagrams also contribute to the decay. They are nonresonant diagrams, similar to that of Fig. 3, in which one of the intermediate gauge bosons is produced away from its mass-shell. In Table 2 we present the values of the branching ratio obtained by using Eq. (6). By comparing these values with the  $t \rightarrow WZb$ branching ratio evaluated by using the convolution method we find that the results are in reasonable agreement. It should be noticed, however, that this agreement is also a consequence of an accidental cancellation between the square of the non-resonant diagrams and the interference among these diagrams and those proceeding through the initial  $t \rightarrow WZb$ decay. Ideally, one would like to consider in the calculation of the decay rate only these three latter diagrams. However, such a prescription does not lead, in general, to a gauge-invariant result.

The decay-chain method has been considered in Ref. [2] to account for the finite-width effects of the W and Z bosons in the  $t \rightarrow WZb$  decay. In that case, the decay rate has been evaluated from the ratio:

$$\frac{\Gamma(t \to b\mu\nu_{\mu}e^{+}e^{-})}{BR(W \to \mu\nu_{\mu})BR(Z \to e^{+}e^{-})}$$
(7)

which differs from Eq. (6) for the choice of the final state. The calculation of  $\Gamma(t \rightarrow b\mu\nu_{\mu}e^+e^-)$  in Eq. (7) requires to take into account a set of 10 Feynman diagrams. The first three diagrams are those proceeding through the initial  $t \rightarrow WZb$  decay. This is the only contribution which is directly related to  $\Gamma(t \rightarrow WZb)$ . Other three diagrams involve the radiation of a Z or a W boson from one of the decay product of the first W. These are non-resonant diagrams in which the intermediate W is produced away from its mass-shell. The last four diagrams are those in which the intermediate Z boson is replaced by a virtual photon. In the region of small invariant masses of the final electron pair, these are the diagrams which give the dominant contribution to the decay rate.

Two observations are worth at this point.



Fig. 3. A Feynman diagram, contributing at tree-level to the  $t \rightarrow b\mu\nu_{\mu}e^+e^-$  decay, which has not been considered in the calculation of Ref. [2].

(i) In evaluating the Feynman diagrams, the authors of Ref. [2] have missed the contribution of the nonresonant diagram shown in Fig. 3. Although numerically small, in the unitary gauge, this contribution is necessary to guarantee a gauge-invariant final result for the decay rate. In addition, in the Standard Model, the  $t \rightarrow b\mu\nu_{\mu}e^+e^-$  decay also receives a tree-level contribution from a set of four Feynman diagrams in which the virtual Z is replaced by a Higgs boson. Also these diagrams have not been taken into account in Ref. [2]. We find that, numerically, the contributions of these diagrams is negligible for values of the Higgs mass larger than approximately 100 GeV.

(ii) In the theoretical evaluation of the  $t \rightarrow$  $b\mu\nu_{\mu}e^{+}e^{-}$  decay rate, all Feynman diagrams contributing to the process, at a given order in perturbation theory, must be taken into account. However, as discussed before, because of the contribution of the virtual photon, with the prescription in Eq. (7) any direct connection between the  $t \rightarrow b \mu v_{\mu} e^+ e^-$  and the  $t \rightarrow WZb$  decays is lost. To show it clearly, we plot in Fig. 4 the differential  $t \rightarrow b \mu v_{\mu} e^+ e^-$  decay rate, as a function of the invariant mass of the electron pair, as obtained either by including in the calculation all Feynman diagrams contributing to the process or only the three diagrams which involve the initial  $t \rightarrow WZb$  decay. Although the latter do not form a gauge-invariant set of Feynman diagrams, the comparison is instructive. The numerical discrepancy, which is seen between the result of the full calculation and the genuine  $t \rightarrow WZb$  contribution, it is mainly determined by the contribution of the virtual photon in the region of small values of the  $e^+e^-$  invariant mass.



Fig. 4. The differential rate  $d\Gamma/d(m_{ee}^2/m_Z^2)$ , for the  $t \rightarrow b\mu\nu_{\mu}e^+e^-$  decay, computed by considering all Feynman diagrams contributing to the process or only the three relevant diagrams which proceeds through the initial  $t \rightarrow WZb$  decay. The top quark mass is fixed to  $m_t = 175$  GeV.

In order to reduce this contribution, the authors of Ref. [2] have introduced a cut on the values of this mass, requiring  $m_{ee} \ge 0.8m_Z$ . As can be seen from Fig. 4, this cut is indeed effective to that purpose, and in the selected kinematical region the  $t \rightarrow b\mu\nu\mu e^+e^-$  decay mainly proceeds through the initial  $t \rightarrow WZb$  decay. However, with this prescription, the definition itself of the  $t \rightarrow WZb$  decay rate becomes dependent on the specific choice of the cut.

As we have shown before, a convenient definition of the  $t \rightarrow WZb$  decay rate, which is independent of any kinematical cut, is provided by the convolution method. The results obtained in this way for the  $t \rightarrow WZb$  decay rate are roughly 3 times larger than those obtained in Ref. [2], the main reason being the introduction of the kinematical cut in the definition of the decay rate. On the other hand, we emphasize that the calculation of Ref. [2] is of interest for the evaluation of the signal and the relevant background rates in the experimental study of the  $t \rightarrow WZb$  decay when the  $b\mu\nu_{\mu}e^+e^-$  is considered as a final state.

#### 4. Conclusion

In this Letter we have computed the  $t \rightarrow WZb$  decay rate, at the leading perturbative order in the Standard Model, by including in the calculation the crucial effects of the W and Z widths. We finds that these effects increase the total decay rate by orders of magnitude, for a top quark mass of approximately 176 GeV, and allow the decay below the nominal threshold.

In the limit in which the finite-width effects are neglected, the results of the previous studies of the  $t \rightarrow$ WZb decay [1–3] are in disagreement among each other. We have repeated the calculation in this limit, and obtained predictions in agreement with those of Ref. [3], but with numerical differences with respect to those of Ref. [1] and, near threshold, of Ref. [2].

We have also shown that a practical definition of the  $t \rightarrow WZb$  decay rate can be provided even in the presence of large finite-width effects of the W and Z bosons. This definition is based on a convolution of the decay rate, computed in the limit of stable particles, with two distribution functions for the invariant masses of the gauge bosons. The distributions are related to the imaginary part of the gauge boson propagator, and are centered around the physical value of the W and Z masses with spread controlled by the particle widths. The results obtained in this way for  $\Gamma(t \to WZb)$  differ, by roughly a factor 3, from those obtained following the procedure of Ref. [2]. The main reason of such a discrepancy is the introduction of a kinematical cut which is included, in Ref. [2], in the definition of the decay rate.

As a final estimate of the  $t \rightarrow WZb$  branching ratio, evaluated by taking into account the finite-width effects, we get:

$$BR(t \to WZb) \simeq 2 \times 10^{-6} \tag{8}$$

for a top quark mass in the range between 170 and 180 GeV. Eq. (8) indicates that, within the Standard Model, the branching ratio is quite small so that the observation of the decay at the LHC is extremely difficult, if at all possible. Alternatively, the observation of the  $t \rightarrow WZb$  decay at the LHC with a larger rate would signal the presence of new physics.

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### References

- R. Decker, M. Nowakowski, A. Pilaftsis, Z. Phys. C 57 (1993) 339, hep-ph/9301283.
- [2] G. Mahlon, S. Parke, Phys. Lett. B 347 (1995) 394, hepph/941225;
  - G. Mahlon, hep-ph/9810485.
- [3] E. Jenkins, Phys. Rev. D 56 (1997) 458, hep-ph/9612211.
- [4] J.L. Diaz Cruz, D.A. Lopez Falcon, Phys. Rev. D 61 (2000) 051701, hep-ph/9911407.
- [5] V. Barger, W.Y. Keung, T.G. Rizzo, Phys. Rev. D 40 (1989) 2274.
- [6] T. Muta, R. Najima, S. Wakaizumi, Mod. Phys. Lett. A 1 (1986) 203.

- [7] W. Beenakker et al., in: Physics at LEP2, Cern Yellow Report CERN-96-01, Vol. 1, 1996, p. 79, hep-ph/9602351.
- [8] G. Lopez Castro, J.L. Lucio, J. Pestieau, Mod. Phys. Lett. A 6 (1991) 3679.
- [9] M. Nowakowski, A. Pilaftsis, Z. Phys. C 60 (1993) 121, hepph/9305321.
- [10] U. Baur, D. Zeppenfeld, Phys. Rev. Lett. 75 (1995) 1002, hepph/9503344.
- [11] J. Papavassiliou, A. Pilaftsis, Phys. Rev. Lett. 75 (1995) 3060, hep-ph/9506417;
  J. Papavassiliou, A. Pilaftsis, Phys. Rev. D 53 (1996) 2128, hep-ph/9507246.