sciendo

CAIM
ISSN 2038-0909

Commun. Appl. Ind. Math. 15 (2), 2024, 44–59 DOI: 10.2478/caim-2024-0012

Research Article

Enhancing Math education for visually impaired students: alternative text implementation in LaTeX, *MathJax*, *MathML* and *LAMBDA*

Giampaolo Rossi[1*], Claudio Fornaro[1]

[1]Facoltà di Ingegneria, Università Telematica Internazionale UNINETTUNO, Corso Vittorio Emanuele II, 39, 00186 Roma, Italy

[*]Email address for correspondence: g.rossi23@students.uninettunouniversity.net

Communicated by Alessandra De Rossi

Received on 06 25, 2024. Accepted on 08 09, 2024.

Abstract

This article outlines an innovative procedure to improve the accessibility of Mathematics for secondary school students with visual impairments. Using LaTeX, a widely used typesetting system, a transformative approach is developed that converts traditional mathematical content into three accessible formats: *PDF*, *MathJax* and *LAMBDA*. Central to this system is the integration of alternative text, which offers full descriptions of images and mathematical formulae and promotes a richer understanding of its content. The broader implications of this project include the introduction of novel teaching models for educators, enhanced accessibility of Mathematics programmes, and the potential to encourage the enrolment of visually impaired students in science degree courses. Ultimately, this work contributes to the creation of the conditions for the development of an inclusive and barrier-free learning environment in Mathematics education.

Keywords: `visual impairment, accessibility, Mathematics, assistive technology, LaTeX, MathJax, MathML, LAMBDA.`

AMS subject classification: `97U70, 97U80.`

1. Introduction

The study of Mathematics for visually impaired students has historically been regarded a challenging issue to address, which has significantly undermined the academic performance, the motivation to study, and ultimately, the access to scientific studies of a multitude of visually impaired students. Distance learning for visually impaired students presents challenges similar to those already encountered by both curricular and support teachers. In this context, it is of the utmost importance to maintain effective interaction at a distance and to develop bespoke teaching solutions that can be used through the digital channels of distance learning. Despite the cessation of the health emergency precipitated by the Covid-19 pandemic, the advent of integrated digital didacticism has become widespread in educational institutions at all levels. This modality must not be considered an ad hoc measure that should be used only in emergencies; rather, it must be integrated into the curriculum as a supplementary mode of instruction, without supplanting traditional classroom teaching. Consequently, it is imperative that teachers and students receive comprehensive, pertinent, and ongoing training. Rather than viewing distance teaching as a failure to share the learning environment, it should be seen as an extension of it. This does not negate the relational value that underlies teaching and learning, even in the case of visually impaired students. In particular, there appears to be a common belief that the lack (or severe impairment) of sight prevents from studying Mathematicsin depth or it makes it too onerous.

Mathematics is less intuitive to be represented than simple text because a one-dimensional string of characters should denote mathematical formulae, even though they are actually two-dimensional constructs. The page is continuously used to display symbols at various levels (exponents, subscripts, fractions, matrices) and also in the form of diagrams and graphs. These are not uncommon or ancillary elements in mathematical texts. If these difficulties are not adequately addressed through the use of assistive technologies, they can create numerous and serious difficulties for visually impaired students who wish to conduct scientific studies as early as secondary school. In addition, they can influence students' choice of university. A student who has not had the opportunity to engage in scientific studies in secondary school is unlikely to choose to pursue them at university.

Consequently, the **primary** objective is to implement the insertion of alternative text directly into the LaTeX source, thereby enabling the generation of differentiated compilations during the processing phase. The automation of the compilations, through the use of simple command-line instructions that can be adapted to different formats, will enable the insertion and display of alternative text in the output. This approach will ensure that the original length of the text is preserved during conversion to *PDF*.

A **further** objective, following the trial run in the chapter on *Natural numbers*, was to complete the insertion of an alternative text of the entire module on *Numerical sets* and the one on *Infinitesimal calculus*, and add them to the *Resources* section, subsection *Matematica Libera* of the portal of the European project *DDMATH* (*Erasmus+*).

2. Context

In recent times, the Italian Agency for the Evaluation of the University System and Research (ANVUR) has published a report on the status of students with disabilities in Italian universities. In particular, the National University Conference of Disability Delegates (CNUDD) working group was involved, and the survey was sent to 90 public, private, and digital Italian universities (out of a total of 98). Furthermore, the survey benefited from input from third-party public bodies (National Institute of Statistics, Ministry of University and Research) and employed procedures that combined a broad and long-term vision with operationality. The data on the distribution of students with disabilities [1], with particular reference to the school year 2020/2021 and focusing on secondary school [2], indicate that students with visual impairments predominantly attend high schools rather than technical or vocational schools, and a considerable percentage enrol in university. It can be reasonably assumed that these students will select a digital university to a greater extent than the average student, due to the services that this type of university makes available to students with special needs. It should be noted that a relatively small proportion of students with visual impairments opt for a degree programme in a scientific field, in comparison to other types of disabilities.

3. Mathematics for visually impaired students

The relationship between blindness and scientific studies (or STEM) is a topic worth further investigation. Although there are notable exceptions, the prevailing sentiment and educational practices appear to be influenced by the belief that a lack of sight prevents or makes the study of Mathematics too strenuous. Upon examination of the scheduling of a Mathematics course of study, it becomes evident that in numerous instances blind students are assigned a less rigorous workload. This is justified by the fact that technical aids do not permit the same speed and precision in the execution of tasks and are not feasible in distance learning. This, in fact, sometimes entails a reduction in the workload or an adjustment of the school's requirements, which sacrifices written communication. Indeed, even important parts of the curricular project are entrusted to oral communication.

4. Two crucial decisions at the base level

The focus has been on making Mathematics accessible to students who are totally blind because, there are several parameters to be considered regarding accessibility and inclusion for blind people in general. Every blind individual has his own unique characteristics and specificities. Some of them are able to operate under particular conditions of contrast or brightness, while others may have a reduced visual field or difficulty focussing on a fixed point. Consequently, adapting this procedure to such disparate conditions is a considerably more complex endeavour. In addition, visually impaired individuals, those with upper limb disabilities, and those who receive DSA have different challenges and requirements.

Most blind students do not exhibit cognitive deficits; rather, they demonstrate normal intelligence. The intelligence capacity is contingent on the availability of a stable variable, namely working memory capacity. This variable is highly correlated with intelligence. Blind people are dependent on their working memory to process information to a much greater extent than those who are sighted. The study corroborates the findings of numerous previous studies that have yielded comparable results: blindness appears to enhance working memory. It is important to note that it is not blindness itself that compensates for a lack of visual information and external visual storage of information [3]; rather, it is the individual and institutional reaction to blindness that compensates for this lack. The primary challenge for these students in learning Mathematics is the difficulty in accessing paper resources (textbooks) and digital resources (e.g. *PDF* documents) with the typical peculiarities of a scientific text (formulas, graphs, specific symbols).

The decision to move from the traditional approach of Mathematics volumes to a modular version was made with the intention of enhancing the flexibility of use for all students. Within the same module, the topics covered are homogeneous, which facilitates the search for a particular topic and potentially improves the ease of learning. Two modules should then be considered: *Numerical sets* and *Infinitesimal calculus*. The two chapters, *From Rationals to Hyperreals* and *Hyperreal*s, are included in the *Numerical sets* module as in-depth chapters and prerequisite chapters under the *Infinitesimal calculus* module.

5. Technologies for the accessibility of Mathematics

The accessibility of Mathematics texts for blind students in secondary education is a significant issue. Currently, there is a dearth of accessible Mathematics texts available to blind students. In the case of non-scientific texts, publishers generally provide blind students with an editable version, which is typically in *MS Word* format. In particular, the scientific texts pertaining to Mathematics are provided in *PDF* format. As a result, such texts are inaccessible to blind students. Nevertheless, new solutions have been implemented in some Italian universities. A notable example is the S. Polin Laboratory in Turin which offers a web page *Biblioteca Accessibile* [4] that provides accessible texts in *PDF* and *HTML* formats. For instance, books for scientific university courses (such as Mathematics, Physics, Engineering, and Economics), books on basic Mathematics, and Statistics for university courses are included. The PDF documents have been produced using the *LaTeX Axessibility* package, the web app *AudioFunctions* and the *LaTeX Axesscleaner* file cleaner. The *HTML* documents were converted from LaTeX using the *LaTeXML* software. To investigate the most functional and suitable technologies and strategies to assist visually impaired people in their scientific studies, the project *"Per una Matematica accessibile e inclusiva"* (*For accessible and inclusive Mathematics*) was started in 2012 within the Department of Mathematics of the University of Turin. In 2018, the project underwent further development with the establishment of the S. Polin Laboratory, which is dedicated to the research and testing of new assistive technologies for STEM. The spectrum of activities has been expanded to include motor and sensory disabilities, and more recently to specific learning disorders [5]. In the present era, people with disabilities and Specific Learning Disabilities (SLDs) are no longer impeded in their ability to read and write well-structured, inline texts (i.e., without graphs or formulas) due to the advent of computers and mobile devices, which have facilitated the use of aids such as speech synthesis, *Braille* bars, and magnifying glasses. In contrast, the full use of educational resources with STEM content (including formulae, graphs, and diagrams) remains an unresolved issue, despite the current considerable technological development [5].

If we accept the definition of accessibility in computer science as "the ability of a device, service, or resource to be easily usable by any type of user" [6], it is necessary to consider the conditions under which individuals with reduced or impeded sensory, motor, or mental abilities intend to use the information systems and software resources available to them, including and especially planning for the use of assistive technologies. Those with visual impairments access textual content through sensory channels other than sight, in particular hearing and touch. They also use assistive technologies that facilitate the use of information through these senses, such as screen readers and *Braille* bars.

A few years ago, under the leadership of Professor Antonio Bernardo, a group of teachers coordinated their expertise to create a Mathematics textbook for the two-year scientific high school: the so-called *Matematica C^3* project. The group made the prudent decision to release the work under a free Creative

Commons (CC-BY-SA) licence, which allows the reproduction and dissemination of the work in any medium and the creation of works derived from *Matematica C^3*.

The development of this project, proved to be particularly promising. *Matematica Libera* represents an evolution of the *Matematica Dolce* project, which itself is derived directly from *Matematica C^3*. *Matematica Libera* has evolved to encompass a broader range of educational institutions, transcending the limitations of traditional scientific schools to encompass a more diverse array of secondary educational institutions. To facilitate this transition of the school context, a specific restructuring is presented in content-related modules, rather than volumes linked to the five curricular school years. The new project starts with the introduction of a new corpus of documents and two new packages that streamline and simplify the preamble of each module. *Matematica Libera* also aspires to cover all topics within the Mathematics curriculum, from natural numbers to integrals. In addition, it provides in-depth studies that can encourage students to analyse specific instances related to Mathematics.

The text is the result of an extensive teaching experience in the classroom and has been modified according to the experience of the direct users. It is not a simple reproduction of books offered by traditional publishers, adapted for visually impaired students. The topics are presented through innovative methodologies that are not typically found in books provided to teachers. These methodologies are derived from critical research and are the optimal method for presenting a topic to students who cannot use their visual sense. This method is rarely aligned with the most common approach. For example, the expression of calculus in $\mathbb{N}$ and the decomposition of factorials are currently proposed using tree graphs (Figure 1). Although graphical methods and visualisations assist some students, they are not accessible to blind students, as the latter can only access the text with the aid of the *Braille* bar.

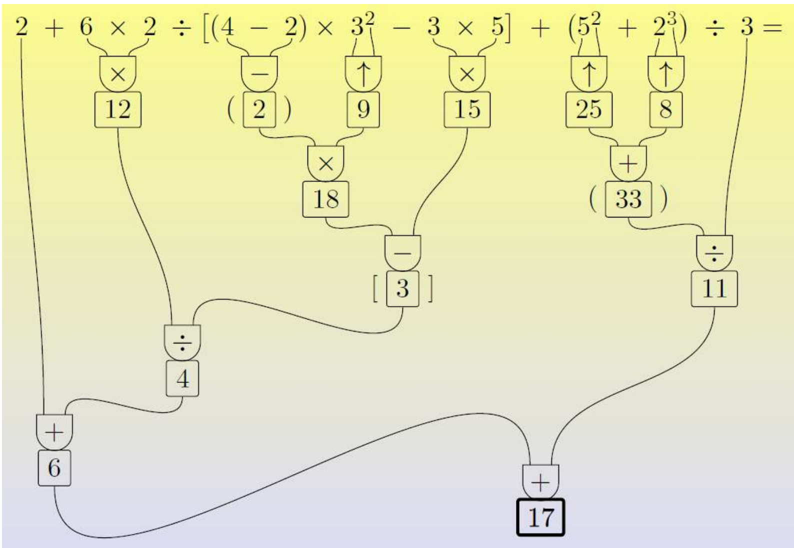


Figure 1. The expression is to be solved using the *tree graph* method.

An alternative method to represent the solution tree, which can also be used with a *Braille* bar, is the so-called linear method [7]. The procedure comprises two steps, which are repeated until the desired result is obtained: (a) highlighting all operations that can be performed; (b) substituting these operations with their result (Figure 2).

The *Matematica Libera* project was developed on the basis of this process, in synergy with a sufficiently solid mathematical and technological basis. The project is stored in a public repository on *Bitbucket*[a]. In order to adapt the text to the specific requirements of the student, a local clone of the project can be created. A version control system (CVS) is then employed to facilitate the synchronisation of local and remote versions and the tracking of all changes made. The project currently encompasses a diverse range of topics and multiple versions of each topic, enabling the creation of customised textbooks. This

[a] Link to the repository of the *Matematica Libera* project: https://bitbucket.org/zambu/matematicalibera/src/master/

$$2+6\times 2\div\left[(4-2)\times 3^2-3\times 5\right]+\left(5^2+2^3\right)\div 3=$$

Underline:	$=2+\underline{6\times 2}\div\left[\underline{(4-2)}\times\underline{3^2}-\underline{3\times 5}\right]+\left(\underline{5^2}+\underline{2^3}\right)\div 3=$
Calculate:	$=2+12\div[2\times 9-15]+(25+8)\div 3=$
Underline:	$=2+12\div[\underline{2\times 9}-15]+\underline{(25+8)}\div 3=$
Calculate:	$=2+12\div[18-15]+33\div 3=$
Underline:	$=2+12\div\underline{[18-15]}+\underline{33\div 3}=$
Calculate:	$=2+12\div 3+11=$
Underline:	$=2+\underline{12\div 3}+11=$
Calculate:	$=2+4+11=$
Underline:	$=\underline{2+4}+11=$
Calculate:	$=6+11=17$

Figure 2. The solution of an expression using the *linear* method.

process entails the use of an existing repository of written materials which are then adapted to align with the specific requirements of the educational curriculum and the individual characteristics of the student population. Furthermore, preparations are being made to create *MathJax* and *LAMBDA* versions of the texts: *MathJax* is an open-source *JavaScript* library that enables the display of *MathML* content on web pages ensuring accurate and accessible presentation of mathematical formulas; *LAMBDA* (Linear Access to Mathematics for Braille Device and Audio Synthesis) is a software system designed to make Mathematics accessible to students who are blind or visually impaired, offering *Braille* and audio output for mathematical expressions. The objective is to integrate with *LAMBDA*, thereby enabling visually impaired students to access content using a screen reader and a *Braille* bar. In this context, accessibility is not merely the passive reception of materials; rather, the underlying paradigm fosters active engagement with Mathematics. This enables students to understand the subject matter through the act of "doing" Mathematics.

6. Instructions for writing new alternative text through LaTeX, *MathML*,*MathJax* and *LAMBDA*

This research project led to the creation of *TeX4ht* code that allows alternative text to be added to the source file and then compiled to generate various output formats, incorporating alternative text whenever needed. Code samples (Figure 3) are available in `matlibera.4ht` file following the path *matematicalibera/deposito/magazzino* in the repository.
The instructions highlighted in Figure 3 are described in [8] and [9].

The **ARIA (Accessible Rich Internet Applications) role** is employed to designate the environment as an image described by the alternative text. *WAI-ARIA* (Web Accessibility Initiative-Accessible Rich Internet Applications) is a technical specification that provides guidance on how to improve the accessibility of web applications. It describes how to add semantics or other metadata to *HTML* content in order to make both user-side controls and dynamic content more accessible. The specification delineates three main features: roles, properties, and states. In the present case, the role ***img*** and the attribute ***aria-labelledby*** were used.

The code that allows the insertion of images and alternative text, if required, is available within the file `matlibera.cls` following the path *matematicalibera/deposito/magazzino*. In order to ensure flexibility and absence of charge, the markup language LaTeX is used [10]. To compile the alternative text, installing the latest version of *TeX Live*, a free cross-platform software distribution for the TeX typesetting system is necessary. Furthermore, *LAMBDA* (v. 2.0) is a system based on the functional integration of a linear mathematical code and an editor for the visualisation, writing, and manipulation of mathematical texts.

```
\NewConfigure{alternativetext}{4}
\renewenvironment{alternativetext}[1]{\a:alternativetext\c:alternativetext#1\d:alternativetext}{\b:alternativetext}

\:CheckOption{blind}
\if:Option

\Hassign\alttext:count=0

\Configure{alternativetext}

{\ifvmode\IgnorePar\fi\EndP

\gHAdvance\alttext:count by 1

\HCode{<div class="alternativetext" role="img" aria-labelledby="alttext-\alttext:count">}}

{\ifvmode\IgnorePar\fi\EndP\HCode{</div>}}

{\HCode{<div class="description" id="alttext-\alttext:count">}}
{\Hcode{</div>}}

\Css{.alternativetext .description{
font-size: 0.9rem;
color: darkgreen;
}}
\else

\Configure{alternativetext}{}{}{\setbox0=\hbox\bgroup}{\egroup}
\fi
```

Figure 3. An extract of the code.

The *Lambda Math Code*, is a direct derivation of the *MathML* code and has been set up for optimal use of *Braille* peripherals and speech synthesis. In real time files are automatically converted into an equivalent version of *MathML* and into one of the most common mathematical writing formats (LaTeX), both in input and output: misinterpretation is thus avoided. This process is described in greater detail in the *DDMATH project - Digital learning in Mathematics* [11]. The file in *lambdabook* format is generated using the *Lambdabook maker* software. The "Libri" menu of *LAMBDA 2.0* enables the opening of books in *lambdabook* format, which is a file obtained by aggregating chapters or sections of a textbook in *lambdax* format (*LAMBDA 2.0* extension).

The portal section includes three versions of each module:

1. the ***PDF*** version does not contain the transcript of the alternative text (this is a single file);
2. the ***MathJax*** version contains the transcription of the alternative text (multiple files for each chapter, organized within a compressed folder);
3. the ***lambdabook*** version contains the transcript of the alternative text (this is a single file).

The alternative text can be added to figures, but can also be used independently. It has the following characteristics:

- the alternative text has a smaller size than the rest of the text, in particular it is typeset in **script** size;
- it is **green** to be easily identifiable by a sighted person (e.g. the teacher) in *MathJax/MathML* files;
- it states "**Alternative text figure:...**" if it is provided for figures; "**Alternative text:...**" in other cases. This is due to the fact that the alternative text in *LAMBDA* is blue if it is within the textual environment, and black if it is within the mathematical environment.

It is of paramount importance to ensure that the compilation process from LaTeX source to *PDF* or *MathJax/MathML* output format is automatic and differentiated.
Three distinct compilations are generated:

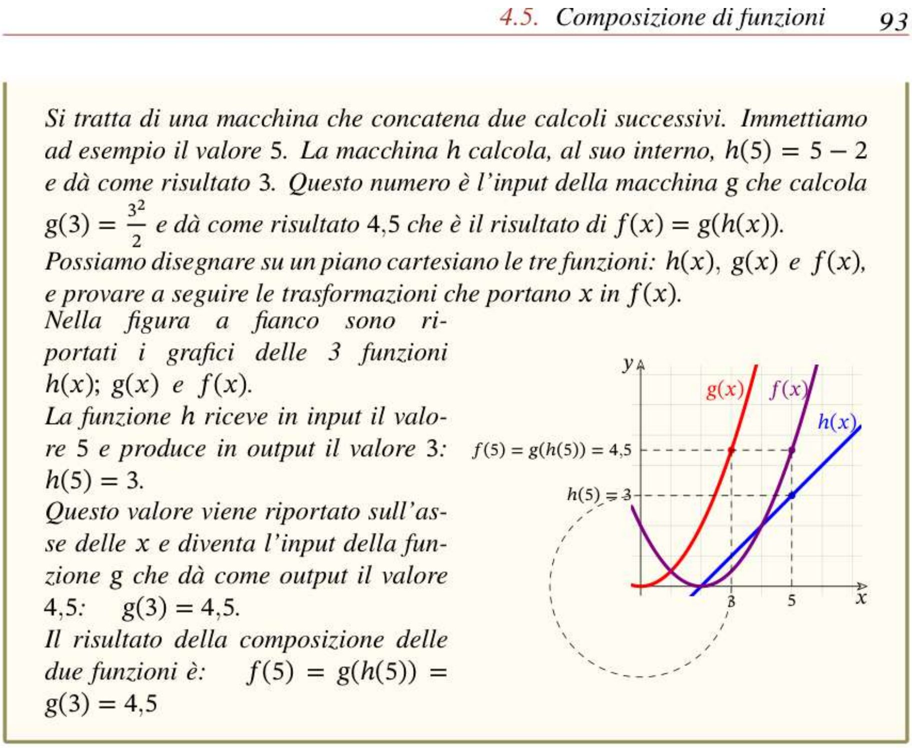
1. for **sighted** students, the alternative text is not displayed in the *PDF* of the entire module generated from LaTeX;

2. for **both sighted and blind** students, the third stage of the process is to convert the LaTeX source to *HTML* by using *MathJax*, so as to generate as many *HTML* files as chapters. In addition, these divisions will be browsable and the alternative text will be displayed;
3. for **blind** students, the LaTeX source text will be converted to *HTML* (*MathML*), which will produce as many *HTML* files as sections. These files are functional to import into *LAMBDA* and the alternative text will be displayed.

As soon as the final compilation is completed, each section file is imported into *LAMBDA*. It is of the utmost importance to verify that the textual content of the book is correctly located within the text environment and accurately transcribed. Furthermore, it is essential to ensure that the mathematical content of the book is correctly positioned within the mathematical environment and rendered accurately within the mathematical notation. Subsequently, each file is saved in *lambdax* format. In *LAMBDA*, the formatting present in the *HTML* file is not maintained. Consequently, the alternative text is not shown in green (as in *HTML*). It is blue if it is in the textual environment or black if it is in the mathematical environment. It should be noted that the font, its size and, more generally, the layout of the text itself, will not necessarily be maintained during conversion. Following this, the *Lambdabook maker* programme requires the addition of each newly created *lambdax* file. As a result, the *Numerical sets* module is generated. Once the process is completed, the resulting file is saved with the extension *lambdabook*. In order to facilitate the use of this software by multiple users, a distributable version should be provided, along with its user manual.

7. Images of graphs of a single or several functions

Consider a *PDF* page as in Figure 4.



4.5. *Composizione di funzioni* 93

Si tratta di una macchina che concatena due calcoli successivi. Immettiamo ad esempio il valore 5. *La macchina* h *calcola, al suo interno,* $h(5) = 5 - 2$ *e dà come risultato* 3. *Questo numero è l'input della macchina* g *che calcola* $g(3) = \frac{3^2}{2}$ *e dà come risultato* 4,5 *che è il risultato di* $f(x) = g(h(x))$.
Possiamo disegnare su un piano cartesiano le tre funzioni: $h(x)$, $g(x)$ *e* $f(x)$, *e provare a seguire le trasformazioni che portano* x *in* $f(x)$.
Nella figura a fianco sono riportati i grafici delle 3 funzioni $h(x)$; $g(x)$ *e* $f(x)$.
La funzione h *riceve in input il valore* 5 *e produce in output il valore* 3: $h(5) = 3$.
Questo valore viene riportato sull'asse delle x *e diventa l'input della funzione* g *che dà come output il valore* 4,5: $g(3) = 4,5$.
Il risultato della composizione delle due funzioni è: $f(5) = g(h(5)) = g(3) = 4,5$

Figure 4. The *PDF* extract shows a graph with three functions.

Teachers or educators can provide an alternative text, such as a link to the graph of the functions under consideration (Figure 5), using the advanced graphical calculator ***Desmos***[b] [12]. This allows blind

[b] A video on the accessibility features of *Desmos* can be accessed via the following link: https://www.youtube.com/watch?v=EqqiqTkThi8

students to mentally explore the graph through audio, for example by listening to the audio description of the crescent, the decrement of the function, or the points of intersection. Once the screen reader has been configured, this process is straightforward.

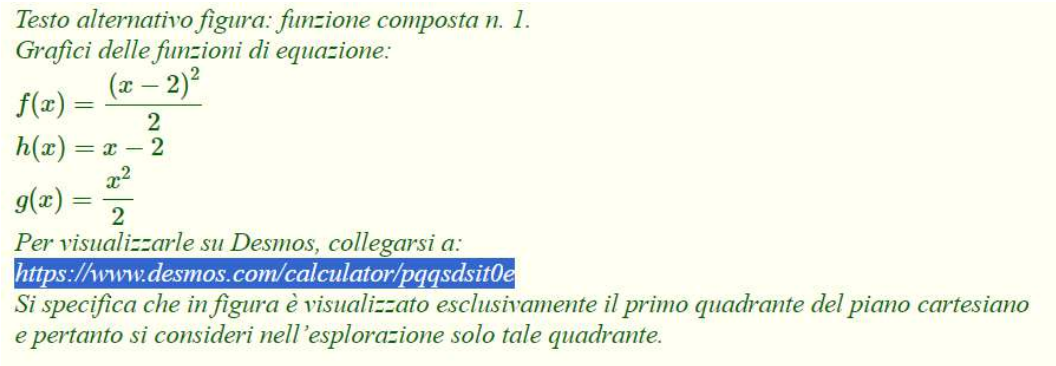

Testo alternativo figura: funzione composta n. 1.
Grafici delle funzioni di equazione:
$$f(x) = \frac{(x-2)^2}{2}$$
$$h(x) = x - 2$$
$$g(x) = \frac{x^2}{2}$$
Per visualizzarle su Desmos, collegarsi a:
https://www.desmos.com/calculator/pqqsdsit0e
Si specifica che in figura è visualizzato esclusivamente il primo quadrante del piano cartesiano e pertanto si consideri nell'esplorazione solo tale quadrante.

Figure 5. The *MathJax* extract of the alternative text contains a link to *Desmos*, which has been highlighted.

Upon opening the link, the content shown in Figure 6 will be visible on the PC screen.

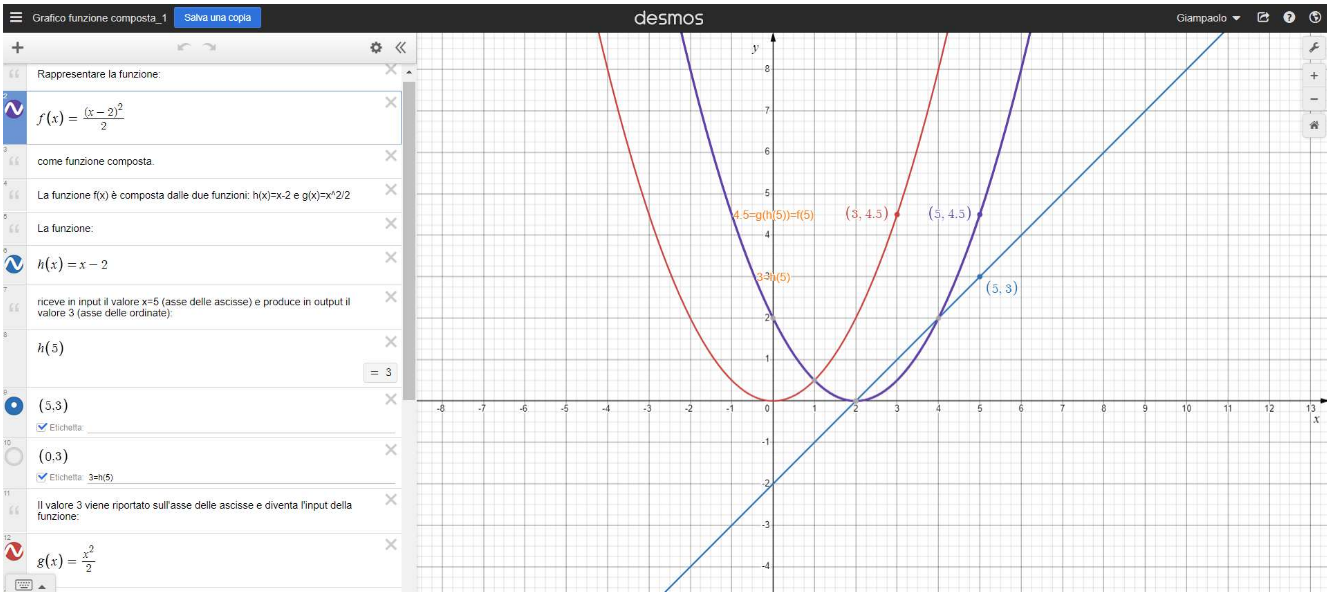


Figure 6. The *Desmos* screenshot displays a graph on the right and comments on the left.

Sonification is a synaesthetic technique that enables the transformation and conveyance of information that is not inherently auditory in nature. This information is conveyed in the form of auditory stimuli [5,13]. The capacity of human hearing to distinguish variations in sound parameters, such as amplitude, frequency, duration, timbre and direction, is exploited to enable the perception of information. Furthermore, the capacity to perceive several sound sources simultaneously and mentally recombine them (polyphony) is also present. This is similar to the experience of listening to an orchestra performing a concert, when the listener hears a multiplicity of instruments playing simultaneously, differing in timbre, power and frequency of sound. The capacity to sonify graphs representing complex data facilitates the interpretation of such graphs by those who are required to read them, and it also provides a novel mode of exploration for individuals with visual impairment [14,15]. The combination of sonification with any additional descriptive alternative text can enhance the accessibility of the function graph device.

8. Issues that emerged during testing

8.1. *Extreme slowness after importing the MathML file into LAMBDA*

The decision to create multiple *HTML* files for each section comes from the need to be able to ensure the module was created in *lambdabook* format to facilitate its use in the *LAMBDA* system. Furthermore, the extreme slowness encountered when modifying the imported *HTML* file, required the creation of the module in *lambdabook* format even when the modification pertained to a single chapter. The software caused slow processing because it requires importing files containing few portions of text (single sections) in order to avoid unmanageable text. Therefore, it was necessary to search for a command for `make4ht` that would split the generated *HTML* file according to the depth of the divisions in the LaTeX file.
The general command is as follows [16,17]:

```
make4ht -d out_directory source.tex "MathJax,3,next,sec-namefile,fn-in,blind"
```

The command converts the `source.tex` file into several *HTML* files containing the mathematical expressions written in *MathJax*. These files are created in the `out_directory`. In particular, the parameters in double quotation marks (in *TeX4ht* they are called *options*) are crucial.

- The **first** option (*MathJax*) adds the fundamental support for the *MathJax* libraries, thus enabling mathematical expressions to be accessed in a navigable format. This can be achieved by displaying them in LaTeX code or by copying them into a text editor for processing (Figure 7).

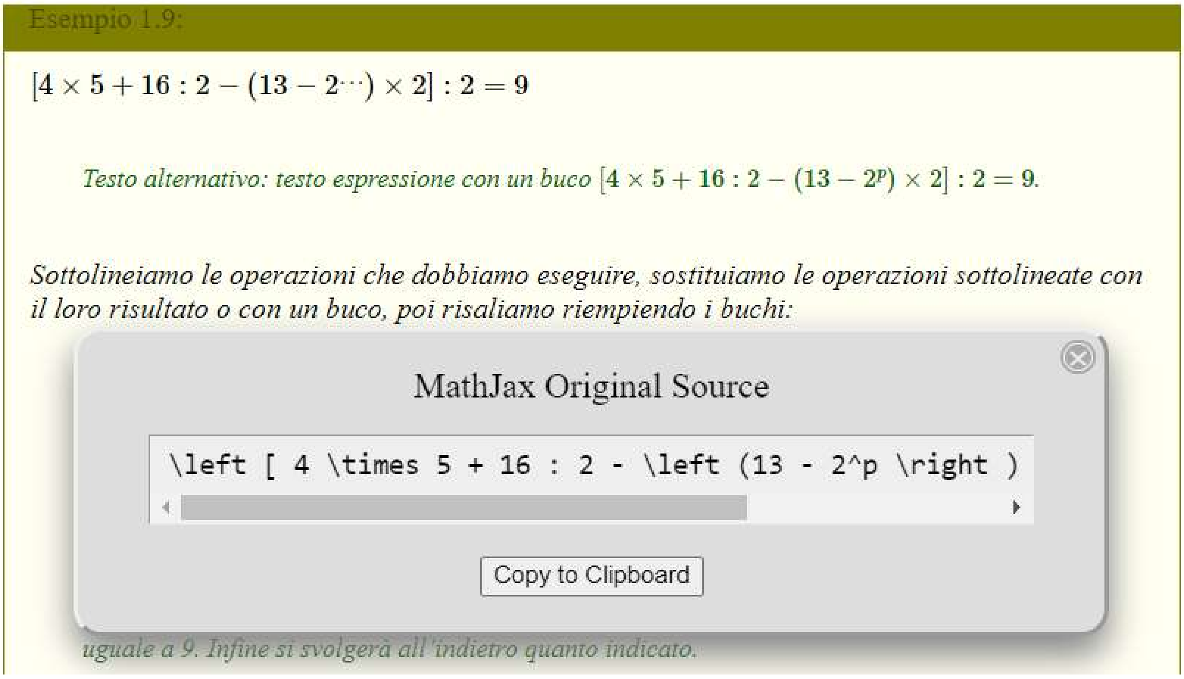


Figure 7. The LaTeX code of a displayed formula in *MathJax* is shown; the alternative text entered is displayed in green on the background.

- The **number 3** specifies the level of divisions for which the derived *HTML* file is broken. This level can be set to 3 at the section level, 2 at the chapter level, or 1, which would result in the file not being broken at all.
- The **next** option has been omitted as some links to move forward or backward in the generated *HTML* pages are already present. The generated *HTML* pages are already present.
- The **sec-namefile** option produces *HTML* file names based on section titles, rather than their numbers.
- The **fn-in** option prints footnotes at the end of each *HTML* page.

- The **blind** option is the crucial one that enables the alternative text to be displayed.

8.2. *Formatting exercise solutions*

It should be noted that a critical aspect was found in both *MathJax* and *MathML* during the testing, relating to the solutions of the exercises. For example, the solution of an arithmetic expression is placed immediately next to the end of the expression itself, in square brackets (Figure 8). When the *HTML* file is imported into *LAMBDA* the same issue arises. Therefore, it could generate ambiguity for a blind student. Figure 9 displays the unambiguous visualisation of solutions for a blind student.

32 *1. Numeri naturali*

1.14. *Calcola applicando le proprietà delle potenze:*

a) $2^5 \cdot 2^3 : 2^2 \cdot 3^6$ $[6^6]$ *e)* $[(3^6 : 3^4)^2 \cdot 3^2]^1$ $[3^6]$
b) $(5^2)^3 : 5^3 \cdot 5$ $[5^4]$ *f)* $5^4 : (3^2 + 4^2)$ $[25]$
c) $[(2^1)^4 \cdot 3^4]^2 : 6^5 \cdot 6^0$ $[6^3]$ *g)* $3^4 \cdot (3^4 + 4^2 - 2^2)^0 : 3^3$ $[3]$
d) $2^2 \cdot (2^3 + 2^2)$ $[48]$ *h)* $\{[(2^3)^2 : 2^3]^3 : 2^5\}$ $[2^4]$

Figure 8. Extract in *PDF* format.

1.14. *Calcola applicando le proprietà delle potenze:*

a) $2^5 \cdot 2^3 : 2^2 \cdot 3^6$ [Sol.: 6^6]
b) $(5^2)^3 : 5^3 \cdot 5$ [Sol.: 5^4]
c) $[(2^1)^4 \cdot 3^4]^2 : 6^5 \cdot 6^0$ [Sol.: 6^3]
d) $2^2 \cdot (2^3 + 2^2)$ [Sol.: 48]
e) $[(3^6 : 3^4)^2 \cdot 3^2]^1$ [Sol.: 3^6]
f) $5^4 : (3^2 + 4^2)$ [Sol.: 25]
g) $3^4 \cdot (3^4 + 4^2 - 2^2)^0 : 3^3$ [Sol.: 3]
h) $\{[(2^3)^2 : 2^3]^3 : 2^5\}$ [Sol.: 2^4]

Figure 9. Extract in *MathJax* format.

8.3. *Multiple columns*

The multi-column display, which is optimal for sighted students, is not functional for blind students (Figure 11). The `matlibera.cls` class (Figure 10) defines a *htmulticols* environment (which behaves like the *multicol* environment) that is rendered inactive in the `matlibera.4ht` file, preventing the generation of multiple columns in *MathML* or *MathJax* (Figura 12).

```
\begin{esercizio}
% \label{ese:1.1}
Rappresenta con grafi le seguenti espressioni:
\begin{htmulticols}{3}
 \begin{enumeratees}
 \item \(57 + 62\) \qquad
 \item \(26 - 7\) \qquad
 \item \(25 \cdot 5\) \qquad
 \item \(48 : 3\) \qquad
 \item \(4^3\) \qquad
 \item \(\sqrt{49}\) \qquad
 \end{enumeratees}
\end{htmulticols}
\end{esercizio}
```

Figure 10. The source file in LaTeX.

1.3. *Rappresenta con grafi le seguenti espressioni:*

a) $57 + 62$ *c)* $25 \cdot 5$ *e)* 4^3
b) $26 - 7$ *d)* $48 : 3$ *f)* $\sqrt{49}$

Figure 11. File in *PDF* format.

1.3. *Rappresenta con grafi le seguenti espressioni:*

a) $57 + 62$
b) $26 - 7$
c) $25 \cdot 5$
d) $48 : 3$
e) 4^3
f) $\sqrt{49}$

Figure 12. File in *MathJax* format.

9. Example of procedure – *Numerical sets module*

1) **Writing the alternative text into the LaTeX source:** the LaTeX and *PDF* files are shown side-by-side on the screen (Figures 13 and 14).

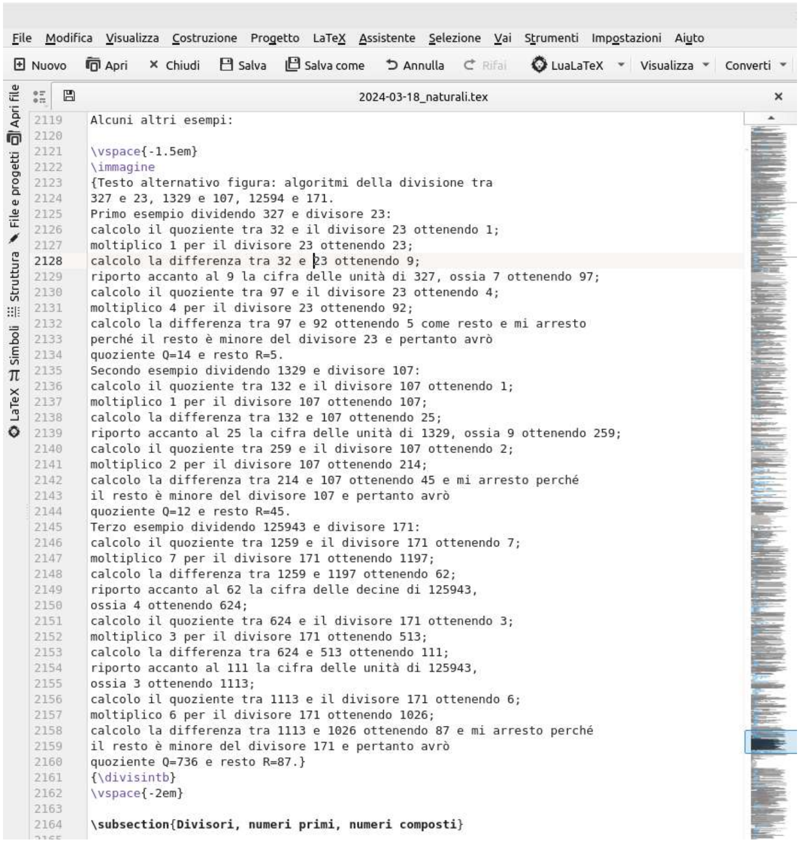


Figure 13. Extract in LaTeX format.

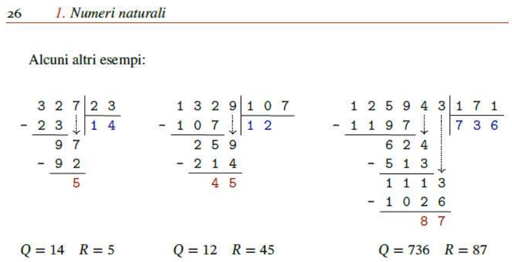


Figure 14. Extract in *PDF* format.

2) **Compilation in *MathJax* with option 2:** all files are saved in the repository within the *html* folder, subfolder *mathjax2*. This option allows the entire module to be explored as linked web pages.
3) **Compilation in *MathML* with option 3:** all files are saved in the repository within the *html* folder, subfolder *mathml3*. This option is only instrumental for importing files into *LAMBDA*.
4) **Import into *LAMBDA* of the *MathML* option 3 compilation files:** once the file has been imported, a check/settlement must be made (Figure 15).
5) **Convertion of the module to book format by using software *Lambdabook maker*:** this step is to be carried out once step 4 has been performed on all files (Figure 16).
6) **Opening of the entire module with *LAMBDA* software:** this allows the user to explore the *Numerical sets* module effectively with the *LAMBDA* software (Figures 17 and 18).

10. Main results

A key result is the successful implementation of the *TeX4ht* code that enables the inclusion of alternative text directly within the source file in LaTeX. This alternative text is then seamlessly integrated into the various output formats generated during compilation, catering to both formats: the one that needs alternative text and the other that does not.

As a result, the text was semplified and integrated, so that the quality of the mathematical content to be learnt by a blind student is equivalent in value and effectiveness to that of a sighted student.

The inclusion of descriptive alternative text within the LaTeX source code enables blind students to

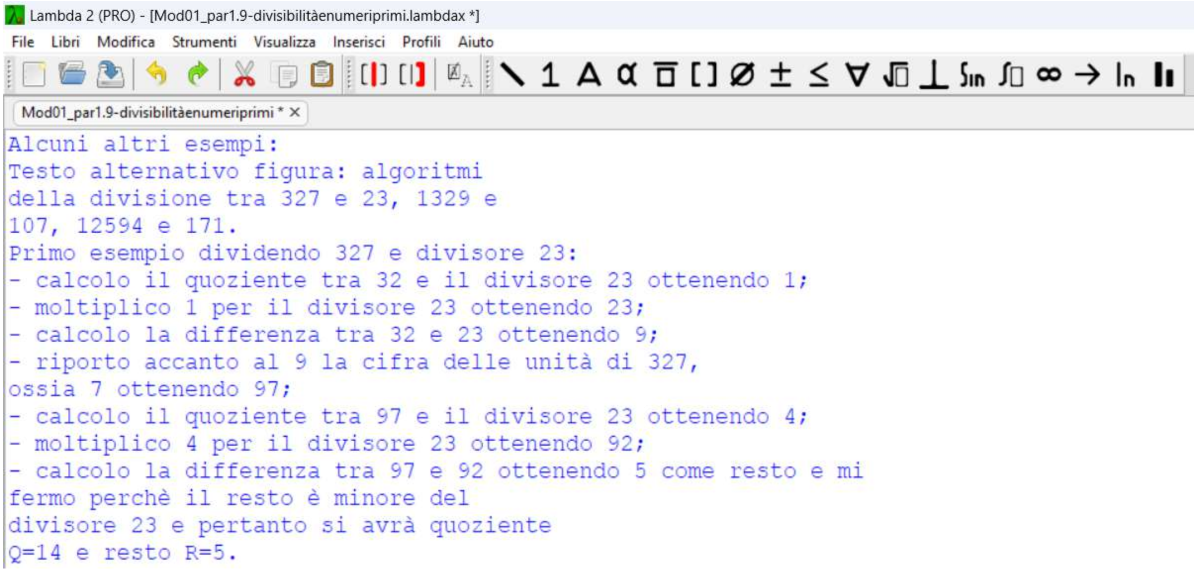


Figure 15. Extract in *LAMBDA* for the 1st example only.

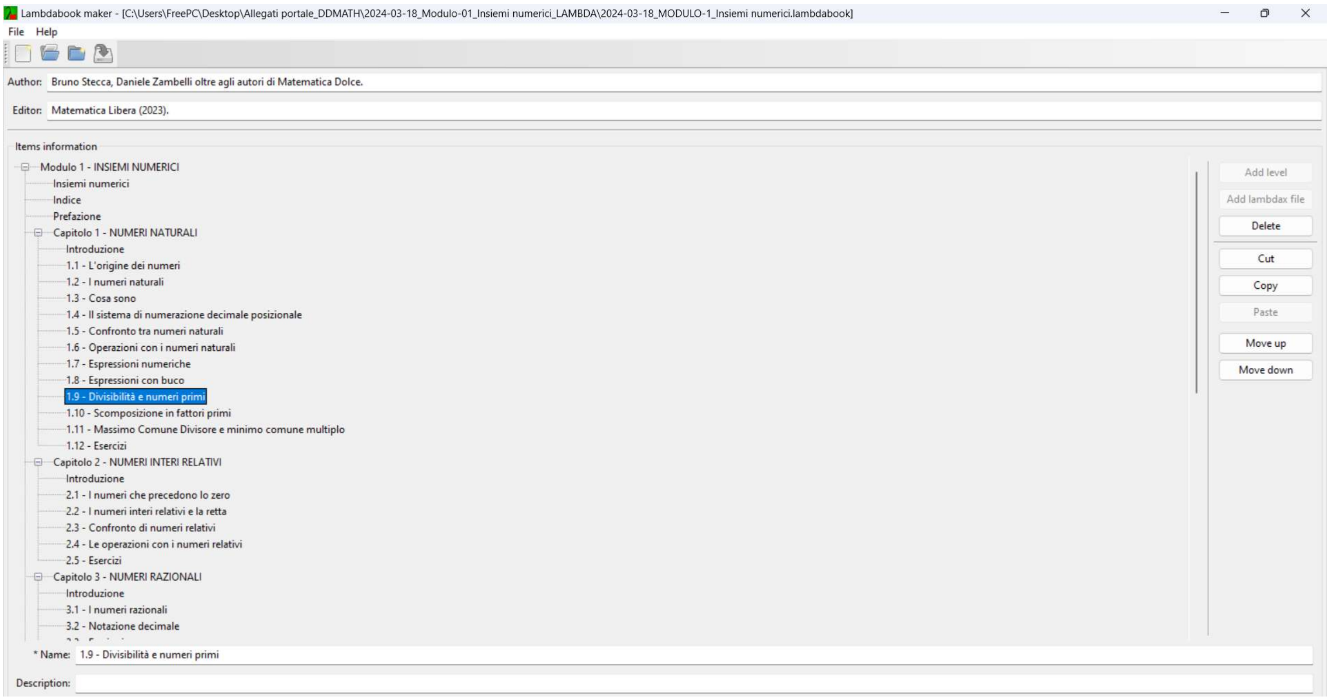


Figure 16. Partial overview of the module with *Lambdabook maker*.

have a comprehensive range of resources for studying and deepening their knowledge of Mathematics and these resources can be used autonomously. Blind students will be able to revise topics covered in class and referred to in textbooks that were previously inaccessible to them. Students will also be able to explore alternative methods for solving problems not covered in class, such as simplifying arithmetic expressions or applying the methods of *Non-Standard Analysis* to *Infinitesimal calculus*.

11. Towards an inclusive Mathematics

The testing phase facilitated the execution and validation of the code designed to compile alternative text into various formats and the verification that the outputs containing the alternative text were

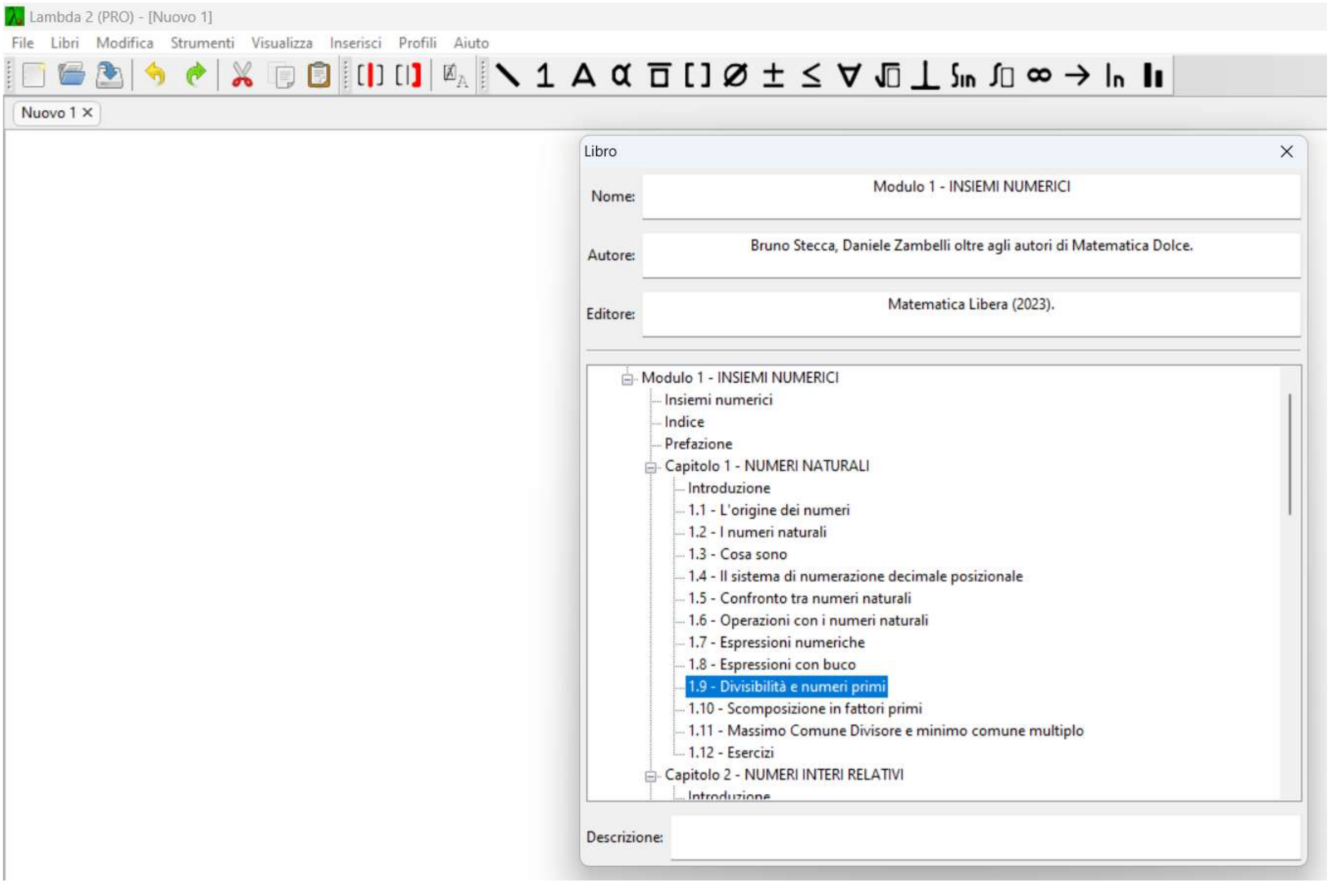


Figure 17. Screenshot in *LAMBDA* of the choice of section to open.

compatible with assistive peripherals for totally blind students (screen reader and *Braille* bar).

The insertion of the alternative text must be carried out by people with solid teaching skills in Mathematics, in order to describe some images to visually impaired students most effectively: in-depth knowledge of the subject matter is required.

It is of the utmost importance to ensure that the compiled alternative text is not merely a caption of the original image but rather provides all the necessary text for a full understanding of the figure for Mathematics teaching purposes. To this extent, it is essential to strike a balance between completeness and essentiality. The capacity to automate the compilation process, providing the required command-line instructions in a straightforward format, and the ability to distinguish between the various formats have a significant practical consequence: the ability to insert and produce the alternative text and to ensure, for instance, that the length of the text is not altered when it is converted to *PDF*.

The integration of the three formats *PDF*, *MathJax* and *lambdabook* into the portal allows the module to be searchable for sighted students without an alternative text. Students will be able to download the file and save it on their personal computer or mobile device.

In the second case (*MathJax* format), students who are blind and have rudimentary LaTeX skills can copy a formula or mathematical expression rendered by *MathJax* into the LaTeX language. This is advantageous in the event that blind students would like to copy the text of an exercise in order to carry it out using their preferred editor or to study a formula in greater detail. Hence, it is possible to paste the LaTeX source code of the formula into *BlindMath*, an accessible LaTeX editor developed for visually impaired people by the *SinAPSi* (Servizi per l'Inclusione Attiva e Partecipata degli Studenti - Services for the Active and Participatory Inclusion of Students) centre of the University Federico II of Naples, which also supports the development of exercises even for those less familiar with LaTeX. The most important

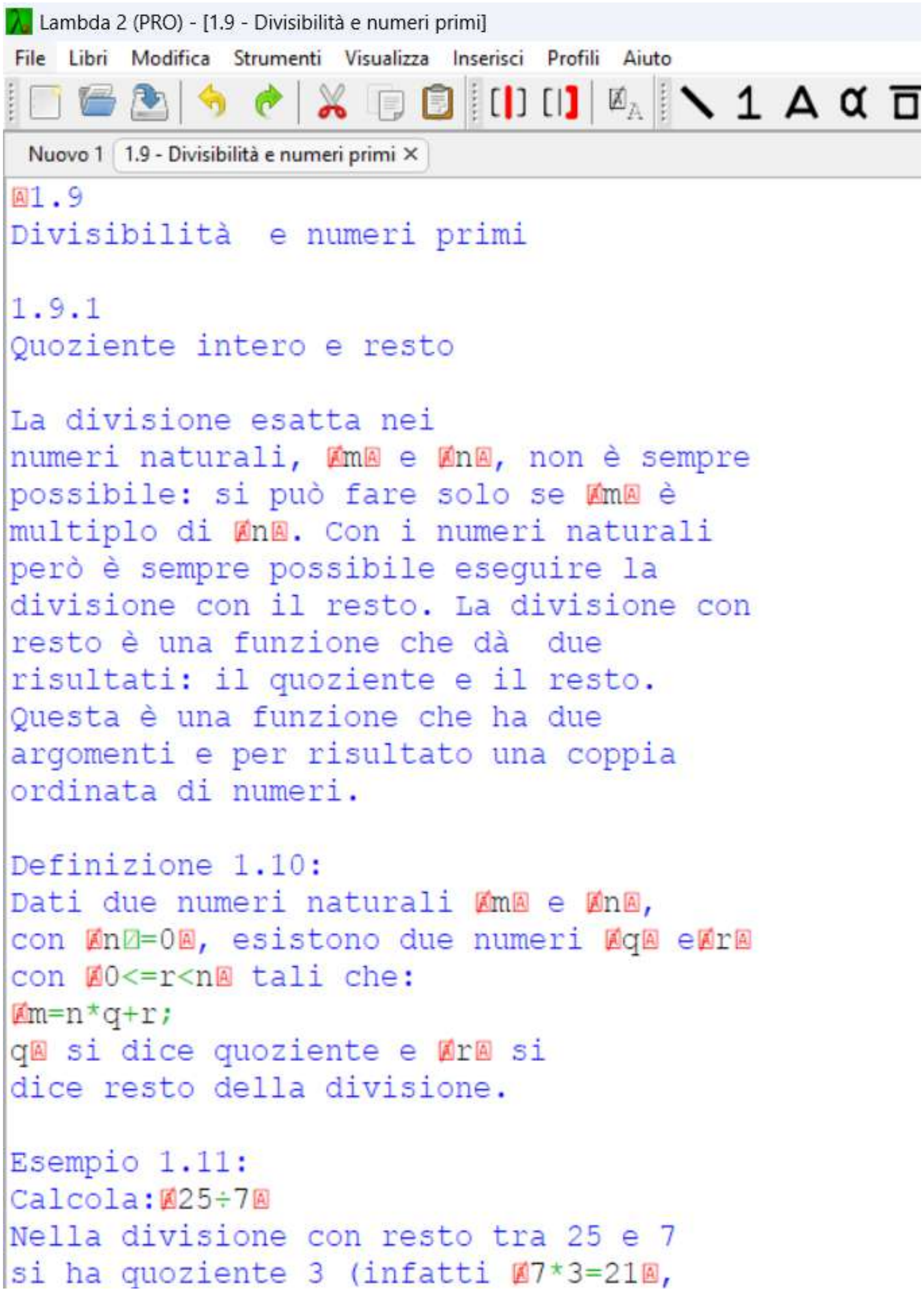


Figure 18. Screenshot in *LAMBDA* once the corresponding section has been opened.

features are the ability to input formulas in a guided manner and the availability of an expression-solving tool. It should be noted that alternative text is also available within the text.

Finally, when using *lambdabook* format, it is possible to navigate the text within the module, browsing through the various chapters and sections as in a book, exploiting the potential of the *LAMBDA* software. This will enable users to read and write Mathematics, as well as manipulate it to solve mathematical expressions, simplify, and develop formulae.

The provision of a comprehensive range of mathematical study materials and additional study resources enables blind students to participate in independent learning. Students can revise and consolidate their understanding of previously covered topics in a format that is accessible to them, even if the original textbook did not include an accessible version for impaired students. In addition, pupils may discover methods to solve problems not analysed in class, such as simplifying arithmetic expressions. They may also investigate topics not introduced in class due to their *Non-Standard Analysis*, such as the treatment of *Hyperreal numbers* within the module of *Sets of numbers* provided as an example.

The expansion of the project will have a positive impact on both blind students and students without severe visual impairments. Blind students will be able to benefit independently from freely accessible teaching resources, while students without severe visual impairments will have the modules available in *PDF* format. Furthermore, the project will support new generations of Mathematics and special education teachers, who will be able to acquire new competencies in accessible Mathematics by experimenting with new models of teaching this subject. In the long term, the adoption of codified procedures for the generation of alternative Mathematics texts is expected to facilitate greater accessibility for visually impaired students, enabling them to pursue degree courses in science and Mathematics without the constraints and apprehension that currently exist.

Acknowledgements

We gratefully acknowledge Professor Giuseppe Nicotra (coordinator of the *DDMATH* project and former Special Needs Teacher at “C. Montanari” High School in Verona) for the opportunity to contribute to the *DDMATH* project and for generously providing us with the *LAMBDA 2.0* and *Lambdabook maker* programmes, which were instrumental in our work. We also extend our sincere thanks to Professor Daniele Zambelli (coordinator of the *Matematica Libera* project and former Mathematics and Physics teacher at the “G. Fracastoro” High School in Verona) for his invaluable assistance in overcoming the technical challenges encountered during the conversion of LaTeX source files into various *HTML* formats. Additionally, we would like to thank Dr. Michal Hoftich, librarian at the Faculty of Education of Charles University in Prague and a prominent developer of free and open-source software (FOSS), for his expert guidance on the use of TeX4ht for alternative text compilation.

References

1. E. Borgonovi, F. Comellini, F. Ferrucci, L. Mason, G. Migliaccio, A. Scaletta, and G. Tanucci, Gli studenti con disabilità e DSA nelle università italiane – Una risorsa da valorizzare, tech. rep., Rapporto a cura dell’ANVUR, 2022.
2. F. Salvini, I principali dati relativi agli alunni con disabilità: AA.SS. 2019/2020 e 2020/2021, tech. rep., Focus a cura dell’Ufficio di Statistica del Ministero dell’Istruzione, 2022.
3. H. Rindermann, A. Ackermann, and J. Nijenhuis, Does blindness boost working memory? A natural experiment and cross-cultural study, *Frontiers in Psychology*, vol. 11, 2020.
4. Laboratorio ”S. Polin”, Biblioteca Accessibile. http://www.integr-abile.unito.it/knowledge-transfer/accessible-library-2/. Retrieved on April 8, 2024.
5. T. Armano, A. Capietto, A. Dragan, C. Bernareggi, S. Coriasco, M. Ducci, C. Magosso, A. Mazzei, M. Nadir, and A. Sofia, Accessibilità dei contenuti digitali per le STEM: un problema aperto. Alcune soluzioni inclusive per l’accessibilità di formule e grafici per persone con disabilità e DSA, in *Atti Convegno Nazionale DIDAMATICA 2020*, pp. 2–11, AICA, 2020.
6. T. Armano, C. Manolino, M. Piroi, M. Borsero, D. Maietta, and A. Capietto, LaTeX tra competenze digitali e accessibilità: un’esperienza di PCTO con il Laboratorio Polin, in *Atti Convegno Nazionale DIDAMATICA 2022*, pp. 118–127, AICA, 2022.
7. M. Bitter, Braille in mathematics education, Master’s thesis, Radboud University Nijmegen, 2013.
8. M. Hoftich, Tex4ht documentation. Chapter 10 – For developers. TeX4ht Project, 2023.
9. ARIA: img role. MDN Web Docs, 2023.
10. R. Giacopini, LaTeX tra i banchi. Possibili applicazioni in ambito scolastico di LaTeX, *Ars TeXnica*, vol. 22, pp. 7–11, 2016.
11. Apprendimento digitale in matematica per studenti non vedenti – Guida al programma LAMBDA 2.0. Progetto DDMATH – Programma Erasmus+, 2022.
12. T. Armano, A. Capietto, D. Maietta, C. Manolino, and A. Sofia, Grafica sonora. Multicanalità e inclusività nella didattica delle STEM, in *Atti DI.FI.MA. 2021 – Apprendimento laboratoriale in*

Matematica e Fisica in presenza e a distanza, pp. 529–532, Università di Torino, 2022.

13. V. Sorge, D. Ahmetovic, C. Bernareggi, and J. Gardner, *Scientific Documents*. Human-computer interaction series, Springer Nature, 2019.
14. S. Balik, S. Mealin, M. Stallmann, R. Rodman, M. Glatz, and V. Sigler, Including blind people in computing through access to graphs, in *ASSETS14 – Proceedings of the 16th International ACM SIGACCESS Conference on Computers and Accessibility*, pp. 91–98, Association for Computing Machinery, 2014.
15. R. Sarkar, S. Bakshi, and P. Sa, Review on image sonification: a non-visual scene representation, *IEEE Transactions on Multimedia*, vol. 14, no. 1, pp. 86–90, 2012.
16. A. Albano, Make4ht, come creare HTLM multi pagina con matematica accessibile, 2022.
17. M. Hoftich, Tex4ht documentation. Chapter 2 – Basic tutorial. TeX4ht Project, 2023.