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Electromagnetic signals produced by elastic waves in the Earth's crust

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Summary. — The paper describes the excitation of geoelectromagnetic-field oscillations caused by elastic waves propagating in the Earth's crust and generated by natural and anthropogenic phenomena, such as earthquakes, explosions, etc. Two mechanisms of electromagnetic signal generation, *i.e.* induction and electrokinetics ones, are considered and a comparative analysis between them is carried out. The first mechanism is associated with the induction of Foucault currents due to movements of the Earth's crust in the core geomagnetic field. The second mechanism is connected with movements of liquids filling pores and cracks of rocks. An equation is derived for describing in a uniform way these two manifestations of seismomagnetism. The equation is solved for body and surface waves. The study shows that a magnetic precursor signal is moving in the front of elastic waves.

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PACS `91.25.Qi` – Geoelectricity; electromagnetic induction and conductivity (magnetotelluric effects).

1. – Introduction

Some fundamental aspects of seismoelectrodynamics, *i.e.* the theory describing electromagnetic fields excited by earthquakes, have been recently reviewed by Guglielmi [1]. In such review, piezomagnetic, inductive, and inertial mechanisms are invoked for the understanding of seismoelectromagnetic phenomena within the framework of a simple model of the Earth's crust.

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In the present paper, induction and electrokinetic mechanisms are analyzed and compared relatively to each other. To do this in a simple and clear way, we do not include in the study the rock magnetic structure (which implies piezomagnetic phenomena) so the magnetic permeability is, of course, unity.

It is known that conducting layers of the Earth's crust moving in the geomagnetic core field during seismic wave propagation induce a variable magnetic field [2]. Such induction seismomagnetic effects have been numerically studied by Mikhailenko and Soboleva [3].

Magnetic-field oscillations could also be generated by electrokinetic phenomena [4]. In fact, the Earth's crust may be considered as a porous moisture-saturated body, and when an elastic wave is propagating in this body it is obvious that the liquid phase moving relative to the solid one sets electrokinetic mechanism in action. In this context, the problem is raised to define the conditions so that this mechanism generates magnetic-field oscillations. Remarkably, vortex motion of the porous medium is the necessary condition to generate magnetic oscillations.

In terms of the real Earth's crust, induction and electrokinetic mechanisms operate simultaneously. As we shall see in the following, the case where an elastic wave is strictly longitudinal presents an exception. In that case magnetic-field oscillations evoke the sole induction mechanism.

Several observations have been collected on this subject in the last thirties. Eleman [5] gave an example of an electromagnetic signal recorded by a helium magnetometer and a telluric experimental survey during seismic wave propagation in the observation site.

Electromagnetic signals were also recorded ahead of the seismic wave front [6, 7]. Observations of seismoelectromagnetic signals in the frequency range 0.1–5 Hz recorded in northern Caucasus were compared with previous ones of similar quality [8].

It has been suggested that the generation of these signals takes place either in the earthquake source [9, 10] or at the seismic wave front [8].

In this paper a simplified equation is derived which describes in a uniform way both the above-mentioned induction and electrokinetic effects. As will be seen, the comparative analysis testifies the fact that induction mechanism prevails at relatively low frequencies and electrokinetic mechanism operates at comparatively high frequencies.

Finally, as a consequence of the quoted mechanisms, the existence of electromagnetic signals preceding seismic waves propagating in the Earth's crust will be shown.

2. – Basic equations of quasi-stationary electrodynamics

To describe seismoelectromagnetic signals the following Maxwell equations in quasi-stationary approximation, and Ohm's law will be used:

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}\,; \qquad \nabla \times \vec{H} = \frac{4\pi}{c}\vec{j}\,; \qquad \nabla \cdot \vec{B} = 0\,; \qquad \vec{j} = \sigma(\vec{E}_{\rm ef} + c^{-1}\vec{v} \times \vec{B})\,. \tag{1}$$

Here, $\vec{E}$ and $\vec{H}$ are the electric- and magnetic-field strengths, respectively, $\vec{B} = \mu\vec{H}$ is the magnetic induction vector (or flux density), μ the magnetic permeability, σ the electrical conductivity, $\vec{j}$ the current density, $\vec{c}$ and $\vec{v}$ are the light and medium motion velocities, respectively. The meaning of $\vec{E}_{\rm ef}$ (effective electric field) in Ohm's law will be specified later (eq. (13)).

The condition for the application of a quasi-stationary approach is defined by the inequality

$$\omega \ll \frac{\sigma}{\varepsilon}\,, \tag{2}$$

where ω is the frequency of the field oscillations and ε is the electric permittivity of rocks. This condition means that the displacement current may be neglected compared with the conductivity current. Inequality (2) is obviously valid for seismic waves.

Equations (1) describe an electromagnetic field under the Earth's surface. Above the Earth's surface the air electrical conductivity is neglected and the second equation of (1) can be substituted by

$$\nabla \times \vec{H} = 0\,. \tag{3}$$

Moreover, with eq. (3) in view, it is implicitly assumed that the wavelength of seismic waves is much less than the distance from the Earth's surface to the ionosphere (about 100 km) as well as the vacuum wavelength c/ω. The latter condition implies that seismic wave velocity is much less than light velocity, this being unconditionally verified with a large margin.

Equations of elasticity should be supplemented by equations of electrodynamics. Two approaches are possible here. Knopoff [11] added into the equation of the solid-body motion a ponderomotive force per unit volume defined as

$$c^{-1}\vec{j} \times \vec{B}\,. \tag{4}$$

As a result, a self-consistent system of magnetoelastic equations, analogous to the system of magnetohydrodynamic equations, was obtained.

However, the following strong inequality:

$$B_0 \ll G^{1/2} \tag{5}$$

is accomplished with a large margin in the Earth's crust, as well as in all known solid bodies.

Here, $B_0 \approx 0.5$ G and $G \approx 10^{11}$ dyne/cm^2 are values of the geomagnetic flux density and of the elastic shear modulus of rocks, respectively. When performing inequality (5), the influence of ponderomotive force (eq. (4)) on the solid-body motion may be neglected and eqs. (1) may be solved in the approximation of the given field $v(x,t)$ of elastic waves.

On the other hand, nothing prevents us from taking the self-consistent system of equations of magnetoelasticity as the basis to seek a solution in the form of magnetoelastic waves, analogous to Alfvén waves.

Then, at a zero-approximation, by the small parameter $G^{-1/2}B_0$, solutions in the form of ordinary elastic waves will be obtained, and in the first approximation perturbations of electric and magnetic fields will be found. In other words, magnetoelastic waves differ from elastic waves in polarization. That is, the magnetoelastic wave propagation is accompanied not only by body deformation but by electromagnetic-field oscillation as well. From this point of view, the problem we are interested in is that of magnetoelastic wave polarization.

However, from the very beginning we shall consider inequality (5) and try to solve the problem at the approximation of the given elastic wave field.

The comparative analysis of induction and electrokinetic mechanisms of electromagnetic-field generation is one of the objectives of our study. That is why to make it more simple and clear we shall ignore the presence of rock magnetic structure and suppose $\mu = 1$, *i.e.* $\vec{B} = \vec{H}$. Somehow, we consider this supposition to be conventional. Simplicity and clarity in expounding the subject are the only reasons justifying this supposition. Real rock masses have magnetic structure. This structure influences piezomagnetic phenomena, underlying a third mechanism of magnetic signal generation produced by seismic waves [12]. We do not consider the piezomagnetic mechanism in this study.

The induction seismological effect is rather strong. At first sight, it is astonishing in how many ways an analogous effect associated with ocean waves and swell is in proportion to the wave amplitude and is the greater the higher is the medium conductivity [13]. With light swell geomagnetic-field disturbance amount to ~ 0.1 nT. Conductivity of the Earth's crust is less than that of ocean water and the typical amplitude of seismic waves is less by many orders than that of ocean waves. But on the other hand, the efficiency of induction mechanism in the Earth's crust is much higher than the analogous one in the ocean.

The induction seismological effect is significant if magnetic-field perturbations are frozen into the Earth's crust and "move" together with seismic waves.

Generally, the "freezing-in" condition for the magnetic field is defined by the Reynolds magnetic number (R_{m})

$$R_{\mathrm{m}} = 4\pi\sigma\xi V_{\mathrm{ph}}/c^2 \,, \tag{6}$$

where ξ is the amplitude of material displacement, σ is the conductivity of the medium, and V_{ph} is the phase velocity of the wave generating the magnetic perturbation. As is well known, for $R_{\mathrm{m}} \gg 1$ the magnetic field can be considered frozen in the medium, whereas the condition $R_{\mathrm{m}} \ll 1$ allows resistive diffusion.

Let us now consider the cases of the sea water and the Earth's crust media. For both these media the condition $R_{\mathrm{m}} \ll 1$ is valid. So, the magnetic perturbation should not be frozen into the substance.

Anyway, in the case of Earth's crust the magnetic-field perturbation can be considered frozen. To illustrate this we shall do a simple consideration. A rough estimate of the geomagnetic-field disturbance can be made through dimensional considerations. In the "freezing-in" regime, amplitude H does not depend on the medium properties. Then, an acceptable combination of the parameters of the problem, which is linear in the seismic wave amplitude ξ, can be rewritten as

$$H \sim k\xi H_0 \,, \tag{7}$$

where k is a wave number.

In the opposite limiting case of diffusion regime similar considerations lead to

$$H \sim R_{\mathrm{m}} H_0 \,. \tag{8}$$

Assuming for example $\sigma = 5 \times 10^8$ s^{-1}, $V_{\mathrm{ph}} = 3 \times 10^5$ cm/s, $T = 10$ s, $\xi = 1$ cm (waves with this amplitude are produced by strong earthquakes) and $H_0 = 0.5$ G, we obtain $H = 0.1$ nT for both eqs. (7) and (8).

Then for studying the propagation of magnetic perturbations in Earth's crust, a specific "freezing-in" condition for the magnetic field is requested. According to this, Gugliemi [14] introduced a δ parameter, defined as follows:

$$\delta = \frac{R_{\mathrm{m}}}{k\xi} = 2\sigma T \left(\frac{V_{\mathrm{ph}}}{c}\right)^2 \tag{9}$$

where T is the period of the magnetic perturbation.

For both cases described by eqs. (7) and (8), $\delta \approx 1$. The magnetic field can be considered frozen into the Earth's crust where $\delta \geqslant 1$. In fact, if for ocean waves $\delta \ll 1$, the Earth's crust "entrains" geomagnetic lines of force stronger than the ocean water does.

3. – A kind of Tolman-Stewart effect in the Earth's crust: the effective electric field and virtual mass

Let us return to the problem of defining the effective electric field E_{ef}. It turns out [15] that during propagation of transverse seismic waves there appears a kind of Tolman-Stewart effect [16] in the Earth's crust, which is considered as a fluid-saturated porous body. As a result, magnetic-field oscillations arise. In the following we shall see that unlike the ordinary electron inertia in metals detected by Tolman and Stewart, the mass of the charge carrier "becomes heavier" in the Earth's crust (see eq. (11)). This enhances the effect and makes it accessible to experimental detection.

It becomes clear that the Tolman-Stewart effect is proportional to the ratio of electron mass m_{e} to its charge $-e$. Then, the effective electric field may be written as

$$E_{\mathrm{ef}} = E + \frac{m_{\mathrm{e}}}{e}\dot{v}\,, \tag{10}$$

where $\dot{v}$ is the acceleration of a metal conductor [16]. So, if we suppose $E_{\mathrm{ef}} = E$, we lose the information on the most important effect.

Our interest in the given phenomenon is caused by the following considerations. It is not the electron, but the ion mechanism of electric conductivity that operates in the Earth's crust, at least in its upper layers. With this in view, the electron mass is to be replaced with ion mass m_{i}. It gains advantage of 1.37×10^5 times, in case the electric conductivity is realized by ions KCl^+.

Anyhow, even with such intensification the Tolman-Stewart effect is small in the Earth's crust. In addition, to evaluate the effect, extreme accelerations have been used, arising during propagation of seismic waves and ripping-up fracture in an earthquake epicentre. But, with the help of some plausible explanations we shall try to make the following statements convincing.

Although the Tolman-Stewart effect is small in itself, still there exists something similar to this effect, and formally speaking the effective electric field should be determined by formula (10) where e is replaced by $-e$ and m_{e} by the effective mass m_{ef} as follows:

$$m_{\mathrm{ef}} = m_{\mathrm{i}} + m_{\mathrm{virt}}\,, \tag{11}$$

where m_{i} and m_{virt} are the mass and virtual mass of the charge carrier, respectively. As defined by the following equation (14), the virtual mass m_{virt} is approximately equal to the mass of a porous fluid, placed in a disk having a radius of about the average pore and a width of about the conductive ion diameter. This definition is referred to relatively narrow pores; in wide pores the radius of the disk approaches the Debye radius.

This new effect, being analogous to the Tolman-Stewart effect, is not so small as $m_{\mathrm{virt}} \gg m_{\mathrm{i}}$, and as mentioned above, in the Earth's crust, considered as a fluid-saturated porous body, conductivity ions, say, "becomes heavier" because of adding virtual mass to the mass of charge carrier.

Let us remind of the origin of formula (10): if the metallic conductor is moving with acceleration, electrons are effected by the inertial force $-m_{\mathrm{e}}\dot{v}$, this being equivalent to the occurrence of an additional electric field $(m_{\mathrm{e}}/e)\dot{v}$. We shall try to realize this idea with reference to ions in an electrolytic solution, filling the pores of the Earth's crust.

Let us imagine an ion as a massive charged particle and let its mass, charge, and radius, be m_{i}, e, and a, respectively. We shall denote density, pressure, and dynamic viscosity of the surrounding liquid as ρ, p, and η, respectively. Since the hard framework is stable we can obtain the force acting on the ions using the Stokes formula of resistance force affecting a spherical body in a viscous fluid flow

$$F = eE - 6\pi a\eta(u - w)\,, \tag{12}$$

where u is the ion velocity in the frame of reference, connected with the hard framework and w is the velocity of the porous liquid surrounding an ion.

If the hard framework is moving with acceleration $\dot{v}$, the ion is affected by an inertial force $-m_{\mathrm{i}}\dot{v}$, apart from that given by eq. (12). Such force is equal to that due to the electric field $-(m_{\mathrm{i}}/e)\dot{v}$.

Till this moment, the course of our discussion is the same that led to formula (10). Now let us consider that, due to the influence of an inertial force, w modifies eq. (12). So, an additional contribution to the effective electric field arises. Really, the porous fluid is moving with velocity w under pressure force $-\nabla p$ relative to the hard framework according to Darcy's law $w = -(k_{\mathrm{i}}/\eta)\nabla p$. The accelerated motion of the framework results in an additional inertial force, $\nabla p = -\rho\dot{v}$ affecting the porous fluid. This additional force will change w for the value $w = -(k_{\mathrm{i}}/\eta)\rho\dot{v}$, where $k_{\mathrm{i}} = k/m$ (k being the permeability coefficient, m the porosity of the hard framework) and $\nu = \eta/\rho$ is the kinematic viscosity of fluid (η being the dynamic viscosity). Then, the term $6\pi a\eta w$ in eq. (12) will be modified into $-6\pi a k_{\mathrm{i}}\rho\dot{v}$.

Taking into account the contributions of the inertial force $-m_{\mathrm{i}}\dot{v}$ and of the resistance force, the force acting on the ion becomes $F = eE - m_{\mathrm{i}}\dot{v} - 6\pi a k_{\mathrm{i}}\rho\dot{v}$.

This is equivalent to the application of an electric field E_{ef} defined as

$$E_{\mathrm{ef}} = E - \frac{m_{\mathrm{ef}}}{e}\dot{v}\,, \tag{13}$$

where m_{ef} is determined by eq. (11) and m_{virt} is given by

$$m_{\mathrm{virt}} = 6\pi a k_{\mathrm{i}}\rho\,. \tag{14}$$

Deriving eq. (14) one can easily see the limitations, even if there is no doubt as to the strong effect in the porous body (resembling the Tolman-Stewart effect).

Let us transform eqs. (13) and (14) so as microscopic parameters e, m_{i}, and a will not appear in subsequent formulae.

First, it should be noted that since the value of k_{i} is equal to the squared averaged size of a pore, it is obvious that $m_{\mathrm{virt}} \gg m_{\mathrm{i}}$ and m_{ef} in eq. (13) could be substituted by m_{virt}.

Second, by rearrangements of formula (14), which completely eliminate the microscopic parameters out of eq. (13), we have

$$\frac{m_{\mathrm{virt}}}{e} = \frac{\varepsilon\zeta}{4\pi\sigma_w\nu}\,, \tag{15}$$

where ε is the electric permittivity of the liquid, $\sigma_w = \sigma/m$ (σ being the electric conductivity) and ζ is the electrokinetic potential (see ref. [17]).

4. – Magnetic signal generation

Eliminating the electric field $\vec{E}$ from eqs. (1) and considering eqs. (13) and (15), we shall obtain

$$\frac{\partial}{\partial t}(\vec{H} + \widehat{\aleph}) = \alpha\nabla^2\vec{H}\,, \tag{16}$$

where

$$\widehat{\aleph} = \beta\widehat{\Omega} + \nabla\times(\vec{H}_0\times\widehat{\xi}\,)\,; \quad \alpha = c^2/4\pi m\sigma_w\,; \quad \beta = c\varepsilon\zeta/2\pi\sigma_w\nu\,; \quad \widehat{\Omega} = (1/2)\nabla\times\vec{v}\,. \tag{17}$$

Here, $\widehat{\Omega}$ is the vorticity, $\widehat{\xi}$ is the displacement vector ($\dot{\widehat{\xi}} = \vec{v}\,$), and $\vec{H}_0$ is the constant external magnetic field.

In deriving eq. (16) it must be considered that the body is homogeneous and stationary, *i.e.* m_{virt} and σ are not dependent either on time or on coordinates (being $\sigma = m\sigma_w$). Equation (16) differs from the ordinary equation of magnetic induction in a moving conductor by term $\beta\dot{\widehat{\Omega}}$. As can readily be seen, this term is proportional to the difference between vorticity of filtration flow of the porous fluid $m\widehat{\Omega}_w$ and vorticity of the hard framework $\widehat{\Omega}$.

We have suggested above that all the parameters of the medium are constant in time and space. Then, as consequence of the continuity equation, the incompressibility condition of motion can be written as

$$\nabla\cdot\vec{v} = 0\,, \tag{18}$$

otherwise porosity m will be variable.

Taking into account eq. (18), we reduce formula (17) to the form

$$\widehat{\aleph} = \beta\widehat{\Omega} - (\vec{H}_0\cdot\nabla)\widehat{\xi}\,. \tag{19}$$

Equation (16) uniformly describes the two seismomagnetic effects under study.

The first term on the right-hand side of eq. (19) represents a new effect connected with vorticity of the porous medium; the second term describes an ordinary effect of electromagnetic induction in a moving conductor.

The ratio between these two terms gives

$$\frac{\beta\widehat{\Omega}}{(\vec{H}_0\cdot\nabla)\widehat{\xi}}\approx\frac{f}{f_\alpha}\,, \tag{20}$$

where f is the characteristic frequency of the phenomenon and f_α is the boundary frequency defined as $f_\alpha = 2\sigma_w \nu H_0/c\varepsilon\zeta$.

The two terms in eq. (19) are the complement of one other. Moreover, eq. (20) shows that the first term dominates at high frequencies ($f \gg f_\alpha$), while the second one prevails at lower frequencies ($f \ll f_\alpha$).

If $\varepsilon = 80$, $\sigma_w = 10^{10}\ \mathrm{s}^{-1}$, $\nu = 10^{-2}\ \mathrm{cm}^2/\mathrm{s}$, $\zeta = 50\ \mathrm{mV}$, $H_0 = 0.5\ \mathrm{G}$, then $f_\alpha = 0.25\ \mathrm{Hz}$.

At higher frequencies ($f \gg f_\alpha$), the magnetic-field generation is described by the following equation:

$$\frac{\partial}{\partial t}(\vec{H}+\beta\widehat{\Omega}) = \alpha\nabla^2\vec{H}\,. \tag{21}$$

4·1. *Body waves.* – Let us consider a longitudinal elastic wave propagating at a velocity c_1 in a homogeneous unbounded medium in the x-direction:

$$\widehat{\xi} \propto \exp\left[ik\left(x - c_1 t\right)\right], \tag{22}$$

where $k = \omega/c_1$ is the wave number, and ω is the wave frequency.

By definition, the displacement vector $\widehat{\xi}$ in a longitudinal wave satisfies the condition $\nabla x\widehat{\xi} = 0$. In the plane wave (eq. (22)) vector $\widehat{\xi}$ is parallel to the x-direction of propagation, that is

$$\widehat{\xi} = (\xi, 0, 0)\,. \tag{23}$$

According to eqs. (22) and (23), vorticity $\widehat{\Omega} = 0$ so, in conformity with eq. (16), only the induction generation mechanism of magnetic oscillations operates. Vector $\vec{H}$ is perpendicular to the x-direction. Without loss of generality, we arrange y- and z-axes so that $\vec{H}_0 = (H_{0\parallel},\, 0,\, H_{0\perp})$. Then, vector $\vec{H}$ has only one component:

$$\vec{H} = (0, 0, H)\,. \tag{24}$$

From eq. (16), and considering eqs. (22), (23), (24), we find

$$H = k\xi H_{0\perp}/(i - kD)\,, \tag{25}$$

where $D = \alpha/c_1$.

The displacement vector in transverse waves satisfies the condition $\nabla\cdot\widehat{\xi} = 0$. Hence, formula (19) can be utilized, vorticity is not equal to zero, and two generation mechanisms are valid simultaneously. However, eq. (16) is linear, so one can analyze these mechanisms irrespectively of one another.

The analysis of the induction mechanism is performed in the same way as for longitudinal waves.

By replacing $c_1 \to c_\mathrm{t}$ and $H_{0\perp} \to H_{0\|}$, where c_t is the velocity of transverse waves, we obtain the result out of eq. (25). In this case vector $\vec{H}$ is polarized as $\widehat{\xi}$. For example, at

$$\widehat{\xi} = (0, \xi, 0)\,, \tag{26}$$

we have

$$\vec{H} = (0, H, 0)\,. \tag{27}$$

For the analysis of the electrokinetic mechanism we shall choose the polarization of transverse waves in the form of eq. (26). There is no loss of generality when considering it. Vorticity equals $\widehat{\Omega} = (0, 0, \Omega)$, where

$$\Omega = \omega^2 \xi / 2c_\mathrm{t}\,. \tag{28}$$

By substituting eq. (28) into (21) we obtain

$$H = i\omega \frac{c_\mathrm{t}}{c} \frac{\varepsilon \zeta m}{\nu} \left[1 - \left(\frac{c_\mathrm{t}}{c} \right)^2 \frac{4\pi i \sigma}{\omega} \right]^{-1} \xi \tag{29}$$

and magnetic oscillations are polarized as follows:

$$\vec{H} = (0, 0, H)\,. \tag{30}$$

4˙2. *Electric-field oscillations.* – A longitudinal elastic wave generates vortex and potential induction (see eqs. (1)). According to eq. (24) the electric field equals $\vec{E} = (0, E, 0)$, where

$$E = (c_1/c) H\,. \tag{31}$$

As far as the potential part of the electric field is excited by a longitudinal wave, the theory we propose does not describe it exactly enough. From equation $\nabla \cdot \vec{j} = 0$, and considering eqs. (13) and (15), we find $\vec{E} = (E, 0, 0)$, where

$$E = \frac{\varepsilon \zeta \omega^2}{4\pi \sigma_w \nu} \xi\,. \tag{32}$$

However, this is not the exact formula as in framing the theory, we did not consider the compressibility of porous fluid, the variation of porosity m in the field of a longitudinal wave, etc. A more correct formula for the potential oscillations of the electric field is reported in Frenkel [18]. It is noteworthy that the limits of our theory do not influence the exactness in defining magnetic-field oscillations.

Being excited by a transverse elastic wave, the electric field has a pure vortex structure. Vector $\vec{E}$ is perpendicular to vectors $\vec{H}$ and $\vec{k}$. Together with them it forms a rectangular Cartesian system of vectors. The amplitude of electric-field oscillations is equal to

$$E = (c_{\mathrm{t}}/c)H\,. \tag{33}$$

5. – Rayleigh waves

A Rayleigh wave is a certain combination of longitudinal and transverse medium displacements [19]. By virtue of that, some difficulties arise at solving a purely mechanical problem, if the porous structure of the Earth's crust is taken into consideration. As stated above, the potential part of the electric field, connected with longitudinal elastic oscillations is not described enough precisely by our theory. As a result, some additional difficulties will arise when proceeding to the problem of electric and magnetic fields generated by Rayleigh waves, propagating along the surface of a porous body.

Therefore, in our case we shall confine ourselves to the analysis of induction generation mechanism only, when the porosity of the body may not be explicitly considered. Porosity m influences latently the induction seismomagnetic signal, as the electrical conductivity σ depends on m.

We shall consider the Earth's crust as a homogeneous elastic body, filling the half-space $z < 0$. Let a Rayleigh wave propagate along the body surface in the x-direction:

$$\widehat{\xi} = \widehat{\varphi}(z)\exp[ikx - i\omega t]\,. \tag{34}$$

Vector $\widehat{\varphi}(z)$ has components (see, for instance, [20])

$$\begin{aligned} \varphi_x &= \kappa_{\mathrm{t}} u \exp[\kappa_{\mathrm{t}} z] + kw\exp[\kappa_1 z]\,,\\ \varphi_y &= 0\,,\\ \varphi_z &= -iku\exp[\kappa_{\mathrm{t}} z] - i\kappa_1 w\exp[\kappa_1 z]\,, \end{aligned} \tag{35}$$

where u and w are displacements and

$$\begin{aligned} \kappa_{\mathrm{t}} &= (k^2 - \omega^2/c_{\mathrm{t}}^2)^{1/2}\,; \qquad \kappa_1 = (k^2 - \omega^2/c_1^2)^{1/2}\,;\\ \omega &= c_{\mathrm{t}} k\nu\,; \qquad u/\omega = -(1-\nu^2/2)(1-\nu^2)^{1/2}\,. \end{aligned} \tag{36}$$

The kinematic viscosity ν is monotonously increasing from 0.874 to 0.955 with the Poisson coefficient increasing from 0 to 1/2 [21].

Outside the body ($z > 0$) the variable part of the magnetic field satisfies the Laplace equation

$$\nabla^2 \vec{H} = 0 \tag{37}$$

and inside the body ($z < 0$) there verifies the induction equation

$$\left(\nabla^2 + p^2\right)\vec{H} = p^2\left[\left(\vec{H}_0\cdot\nabla\right)\widehat{\xi} - \vec{H}_0\left(\nabla\cdot\widehat{\xi}\right)\right], \tag{38}$$

where $p^2 = 4\pi i\sigma\omega/c^2$.

The condition of solenoidality

$$\nabla \cdot \vec{H} = 0 \tag{39}$$

should be added to eqs. (37) and (38).

Field $\vec{H}$ should be continuous at the boundary of the body ($z = 0$) and it vanishes at $|z| \Rightarrow \infty$.

After substituting eq. (34) into (38) we obtain

$$\begin{aligned} \left(\partial^2 - q^2\right) H_x &= p^2[u\kappa_{\rm t} T(z) + w\kappa_1 L(z)]\,, \\ \left(\partial^2 - q^2\right) H_y &= -ip^2 w \left(k^2 - \kappa_1^2\right) H_{0y} \exp[\kappa_1 z]\,, \\ \left(\partial^2 - q^2\right) H_z &= -ip^2[uT(z) + wL(z)]k\,, \end{aligned} \tag{40}$$

where

$$\begin{aligned} T &= (ikH_{0x} + \kappa_{\rm t} H_{0z}) \exp[\kappa_{\rm t} z]\,, \\ L &= (i\kappa_1 H_{0x} + kH_{0z}) \exp[\kappa_1 z]\,, \\ q^2 &= k^2 - p^2\,, \\ \partial &\equiv \partial/\partial z\,. \end{aligned} \tag{41}$$

The dependence of the form $\exp[ikx - i\omega t]$ on time and x-coordinate is supposed but not explicitly written. Equations (40) should be supplemented by the relationship

$$H_x = (i/k)\,\partial H_z \tag{42}$$

which follows from eq. (39). Considering eq. (37) and the boundary condition at infinity, relation (42) is expressed ($z > 0$) as

$$H_x = -iH_z = \text{const} \cdot \exp[-kz]\,. \tag{43}$$

Here we assume that σ does not depend on z. For a more realistic σ profile see [22].

First we shall consider equations for H_x and H_z. We solve them by the method of arbitrary constants variation. As a result we obtain the amplitude of magnetic-field disturbance on the body surface ($z = 0$):

$$H_x = \left[\frac{uT(0)}{q + \kappa_{\rm t}} + \frac{wL(0)}{q + \kappa_1}\right] \frac{p^2 k}{q + k}\,, \tag{44}$$

where $q = \left(k^2 - p^2\right)^{1/2}$, $\operatorname{Re} q > 0$, and, obviously, $H_z = iH_x$.

The dependence on t, x, and z at $z \geqslant 0$ is clear. At $z < 0$ the dependence on z has an awkward form, and it is not presented here (see, for instance [23]).

By analogy, we find H_y. In this case the displacement current $c^{-1}\partial E/\partial t$ at $z > 0$ should be considered. Having made this, one can see that H_y is about by (σ/ω) times less than H_x and H_z.

Electromagnetic-field components may be assembled into two different groups: H_x, H_z, E_y and E_x, E_z, H_y. The corresponding oscillations are to be naturally called oscillations of magnetic and electric type, respectively. One should emphasize that both the two types of oscillations are dependent: all six components form the unified structure of the electromagnetic field of the Rayleigh waves. The electric component of magnetic-type oscillations at $z \geqslant 0$ is equal to

$$E_y = i\nu\,(c_{\mathrm{t}}/c)\,H_x\,. \tag{45}$$

Formula (45) is valid in a rest frame of reference. In case of a frame of reference connected with body surface oscillations, one should add a corresponding kinematic term to the right-hand side of eq. (45).

Remarkably, the polarization relationships (43) and (45) could be used to recognize the induction seismomagnetic signal in case of noises (see also [22]).

6. – Magnetic signals precursors to elastic waves

There exists an analogy between the problem of the magnetic structure of an elastic wave front in a conducting body and the problem of Cherenkov generation of whistlers. Before the front comes out to the body surface, there arises a kind of transitional radiation [24].

To study these phenomena, let us consider a plane elastic wave with a sharp fore front, propagating in a homogeneous conductive medium along the x-axis. Let us suppose that at the moment $t = 0$ the displacement field of the medium be

$$\begin{aligned} \xi(x) &\propto e^{ik_0x} \qquad \text{at } x < 0\,, \\ \xi &= 0 \qquad\qquad \text{at } x \geqslant 0\,. \end{aligned} \tag{46}$$

6·1. *Longitudinal elastic waves.* – To be more precise, first let us consider longitudinal waves. Then, the displacement vector $\widehat{\xi}$ has the form $\widehat{\xi} = (\xi, 0, 0)$ and $\xi = \xi(x - c_1t)$. Only the inductive mechanism operates in this case and eq. (16) becomes

$$\dot{H} - \alpha H'' = -H_{0\perp}\dot{\xi}'\,, \tag{47}$$

where dot and prime mean differentiation with respect to time and coordinate, respectively. Polarization in magnetic signals is given by relationship (24).

Let us convert the displacement field expressed by eq. (46) into the Fourier integral on coordinate x. The Fourier component with the wave number k is equal to

$$\xi_k = \frac{i\xi_0}{2\pi}\int\limits_0^{\infty} e^{i(k-k_0)x}\mathrm{d}x\,, \tag{48}$$

where ξ_0 is the wave amplitude. Let us consider the field $H(x,t)$ as a superposition of propagating plane waves:

$$H(x,t) = \int\limits_{-\infty}^{\infty} H_k e^{ik(x-c_1t)}\mathrm{d}k\,. \tag{49}$$

Component H_k is expressed through ξ_k with the help of eq. (25).

By substituting eq. (48) into (25) and then into (49) we find

$$H(x,t) = \frac{H_{0\perp}\xi_0}{(k_0 D - i)D} e^{ik_0(x-c_1 t)} \qquad \text{at } x < c_1 t\,,$$

(50)

$$H(x,t) = \frac{H_{0\perp}\xi_0}{(k_0 D - i)D} e^{-\frac{1}{D}(x-c_1 t)} \qquad \text{at } x > c_1 t\,.$$

One can see from eqs. (50) that for $x > c_1 t$ there exists a magnetic precursor in front of the elastic wave, exponentially attenuating with increasing distance from the front. The attenuation constant D (distance at which the signal decays by e times) is defined as

(51) $$D = c^2/4\pi\sigma c_1$$

and depends on the medium electrical conductivity σ, on the elastic wave velocity c_1, and does not depend on the frequency $\omega_0 = c_1 k_0$. Behind the front ($x < c_1 t$) the magnetic field oscillates with frequency ω_0.

6˙2. *Transverse elastic waves.* – The analysis of transverse waves is performed in the same way as for longitudinal waves. In this case both induction and electrokinetic generation mechanisms of magnetic precursors operate.

The induction signal is described by eq. (50) when substituting $c_1$with c_t and $H_{0\perp}$ with $H_{0\|}$. This signal is polarized as $\widehat{\xi}$ (*e.g.*, eqs. (26) and (27)).

The magnetic signal of electrokinetic origin is perpendicular to vectors $\widehat{\xi}$ and $\vec{k}$ (see, for example eqs. (26) and (30)). It has the same form as eq. (50) with the following substitutions: $c_1 \to c_\mathrm{t}$, $H_{0\perp} \to -i\omega_0\beta/2$. The value of this signal does not depend on the external magnetic field at all.

Although the result is explicit, it is useful to examine the mechanism of the precursor generation as a phenomenon itself.

First, we shall simplify the source by taking $\xi \propto \delta(x)$ at $t = 0$ instead of conditions (46). Then, $H(x,t)$ is proportional to the following integral:

(52) $$\int_{-\infty}^{\infty} \frac{k\mathrm{d}k}{i - kD} e^{ik(x-c_1 t)}\,.$$

Let us consider it in a complex plane. The only singularity of the subintegral function is placed on an imaginary axis in the upper semi-plane. It contributes to integral (52) only at contour closing of the infinitely far upper half-circle, that being admitted at $x > c_1 t$, *i.e.* before the front of the elastic wave.

This gives

(53) $$H \propto \exp\left[-\left(x - c_1 t\right)/D\right]$$

before the front, as in eq. (50). Behind the front ($x < c_1 t$) the integration contour is closed by the lower semicircle. This gives $H = 0$, *i.e.* unlike that reported by eqs. (50), the magnetic-field disturbance vanishes just after the impulse passes through.

One can see a paradox that the diffusion wave $H(x,t)$, given by eq. (47), propagates on the right of the delta-shaped source, simulating the elastic wave front and does not propagate on its left. Naturally, it is connected with the source motion. As a matter of fact, we are confronted with a kind of Cherenkov generation of magnetic precursor [25]. A more complete understanding of the problem calls for further explanation.

As usually happens, the wave excited by a uniformly moving source, is of diffusive and not oscillating character. Besides, the fact that Cherenkov radiation advances the source and is not behind it, also needs explanation. To clarify, we shall introduce the following analogy.

The dispersion relation for eq. (47), without its right-hand side, looks like

$$\omega = -i\alpha k^2 \, . \tag{54}$$

We shall consider the waves with analogous frequency dependence on the wave number:

$$\omega = \gamma k^2 \tag{55}$$

but with a real proportion coefficient of γ. This dispersion takes place, for example, for whistlers propagating in a plasma along the external magnetic field. The condition of Cherenkov excitation by a source of the type $\delta(x-ut)$ is as follows:

$$\omega = ku \, . \tag{56}$$

This condition, in combination with eq. (55), leads to the resonance condition

$$k = u/\gamma \, . \tag{57}$$

Since we are considering a one-dimensional case, the problem of the Cherenkov cone does not arise.

The radiation advances the source, when the group velocity $v_{\mathrm{g}} = \partial\omega/\partial k$ is greater than the phase velocity $v_{\mathrm{ph}} = \omega/k$, or it is behind the source, when $v_{\mathrm{g}} < v_{\mathrm{ph}}$. If ω is in square dependence on k, we have $v_{\mathrm{g}} = 2v_{\mathrm{ph}}$ and the signal advances the source.

Now by combining eqs. (54) and (56) and supposing $u = c_1$, we obtain

$$k = ic_1/\alpha \tag{58}$$

which is analogous to eq. (57).

It is clear that eqs. (57) and (58) are the poles of a subintegral expression similar to (52) for the waves dispersion laws (55) and (54), respectively. In the case of eq. (55) we have only to define exactly the rule of by-passing the pole (57). By considering in eq. (55) an infinitely small absorption one can verify that the pole $k = u/\gamma$ is to be by-passed from below. Therefore, as in the case of eq. (54), the signal with frequency given by eq. (55) is present in the front of the delta-shaped source and is absent behind it. Thus, the problem of the magnetic structure of the elastic wave front is analogous to that of the Cherenkov generation of whistlers.

All the above-mentioned considerations refer to longitudinal waves, but it is clear that they apply to transverse waves as well.

For practical reasons it is interesting to know how the magnetic precursor comes out to the Earth's surface. In the epicentre location we may consider the elastic wave front to be parallel to the earth-air boundary. Let this boundary be in the plane $x = 0$, with the Earth being in the lower half-space and the air in the upper half-space.

An elastic wave of the form

$$\xi \propto \delta(x - c_1 t) \tag{59}$$

propagates upwards from below. We shall be satisfied with the particular case of eq. (59), as the structure of the magnetic precursor does not depend on medium oscillations outside the front.

We know the magnetic impulse, excited by the source described by eq. (59) in an infinitive medium (see eq. (53)). Let us transform it into the Fourier integral:

$$\begin{aligned} H(x,t) &= \int_{-\infty}^{\infty} H_\omega(x) e^{-i\omega t} \mathrm{d}\omega\,, \\ H_\omega &= \frac{H_{\max}}{2\pi}\left(i\omega + \frac{c_1}{D}\right)^{-1} e^{i\omega x/c_1}\,, \end{aligned} \tag{60}$$

where $H_{\max}$ is the magnetic-field perturbation in the wave front. Free waves moving from the boundary should be added to eqs. (60).

At $x < 0$ the wave propagates downwards:

$$H_\omega^- = A e^{-ipx} \tag{61}$$

and at $x > 0$ it propagates upwards:

$$H_\omega^+ = B e^{+iqx}\,, \tag{62}$$

where $p = (1 + i)/\delta$, $q = \omega/c$, $\delta = c/(2\pi\sigma\omega)^{1/2}$.

Coefficients A and B are taken from the continuity condition of the field at the boundary. As a result we obtain

$$H = 2(|x|/D) H_{\max} \exp[c_1 t/D] \tag{63}$$

with $t < x/c_1 < 0$. Here, $c_1 \ll c$ is taken into account and corresponding small terms are neglected. With $x = 0$ these terms are accounted, but the magnetic signal is small as c_1/c. With $x > 0$ the signal remains as small as with $x = 0$.

We shall evaluate the distance D of eq. (51) at which the signal decays by e times. If $c_1 = 6$ km/s, $\sigma = 10^8$ s^{-1}, then $D = 12$ km. If $\sigma = 10^7$ s^{-1} the value of D exceeds the width of the Earth's crust.

7. – A comparison with observations

By basing on the theory developed in the previous sections, we shall try to explain the magnetic-field response to one of the strongest seismic events ($M = 8.6$): the Alaska earthquake occurred on March 28, 1964 at 03:36:10 UT in the northern part of Prince

Williams bay, with epicenter coordinates 61°6′N, 147°48′W. The ripping-up of rocks began at a depth of 20–30 km.

The seismomagnetic signals, as reported by Eleman [5], were recorded by a helium magnetometer installed in Bergen Park, Colorado. Geographical coordinates of the observation point are: 39°42′N, 105°22′W. Magnetic oscillations, with characteristic period $T = 20$ s and amplitude $H = 0.2$ nT, began at the moment of the surface waves arrival. The following values of seismic wave parameters were obtained: vertical oscillation velocity of 0.4 cm/s, horizontal wave velocity of 0.7 cm/s. This corresponds to displacement amplitudes: $\xi_x \cong 1.3$ cm, $\xi_z \cong 2.2$ cm. Here the x-axis, as earlier, is oriented in the direction of the wave propagation.

The horizontal and vertical components of the geomagnetic field at the observation point were equal to $H = 0.22$ G and $Z = 0.52$ G, respectively. The angle between the geomagnetic meridian and the direction of wave propagation is equal to $\vartheta = 48°$. Then, $H_{0x} = H \cos\vartheta = 0.147$ G and $H_{0z} = Z = 0.52$ G.

To make calculations according to our theory, one should know some other parameters whose exact values are not known. Therefore, we shall assume typical values as follows: $\nu = 0.92$, $c_{\mathrm{t}} = 3 \times 10^5$ cm/s, $c_1 = 5 \times 10^5$ cm/s. An estimate of the magnetic signal amplitude is performed by formula (44) within the "freezing-in" condition, when it produces the maximum possible amplitude value. It should be considered that the instrument at Bergen Park recorded the module of the total vector of the oscillating geomagnetic field. Having data on ω, ξ_x, ξ_z, H_{0x}, H_{0z}, by some simple calculations, we find $H \cong 0.1$ nT.

We have estimated the contribution of the induction mechanism in forming seismomagnetic signals. According to the theory, such signals have amplitude, which is the half of those of many other recordings obtained at Bergen Park. This difference cannot be removed by considering the electrokinetic mechanism, as the oscillation frequency is lower than the boundary frequency f_d.

Nevertheless, we shall make estimations by formula (44) with the condition $\delta \sim 1$ (see eq. (9)). If parameter values are typical, we obtain $H \sim 5 \times 10^{-2}$ nT.

Even if it is correct, still there is a difference of 5×10^{-2} nT. One could explain it by piezomagnetic oscillations, however, it should be admitted that the theory explains the effect only on a qualitative basis. A more detailed analysis considering real conditions of the experiment is needed.

Now we shall consider another type of observations that are concerned with magnetic signals advancing the seismic wave front.

Gokhberg *et al.* [7] reported the case of a weak magnetic signal ($\sim 10^{-3}$ nT), that advanced the seismic wave front by 30 s for an epicentre distance of 200 km and a focal depth of 30 km. Belov *et al.* [6] have revealed a magnetic signal with an amplitude of 0.4 nT, that has advanced the elastic wave by some seconds. The earthquake had magnitude $M = 6.0$ and took place in Kamchatka. Considering a rather small distance between the observation point and the epicentre, one could try to interpret the magnetic signal as an element of elastic wave front. The conclusion seems obvious that the screening effect of the earth-air boundary does not allow observing on the ground the magnetic signal advancing the plane elastic wave front. Since Belov *et al.* [6] observed the signal before the elastic wave front came out to the surface, the curvature of the front is not to be considered. However, it cannot be excluded that the signal was excited not before but after its coming out to the surface.

8. – Conclusions

This study presents a comparative analysis of induction and electrokinetic mechanisms of electromagnetic signals generation associated with the elastic wave propagation in the Earth's crust.

An equation is derived, describing in a uniform way how the two mechanisms operate, and it is shown that induction (electrokinetic) mechanism dominates at comparatively low (high) frequencies.

The basic equation is solved in the form of body and surface waves. It has been demonstrated that a magnetic precursor is moving before the elastic wave front.

The theory developed in the present paper may be of some help in understanding seismoelectromagnetic phenomena and may be utilized to justify observations. In particular, the expounded theory could be used as a basis when discussing the magnetic impulse generation mechanisms, which are often observed on the occasion of earthquakes occurrence and/or natural or man-made explosions [26, 27].

* * *

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