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The 2-variable truncated Tricomi functions

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Abstract

In this paper, we introduce 2-variable truncated Tricomi functions$h_n(x,y)$ and 2-parameter 2-variable truncated Tricomi functions $h_{n,\nu}^{(\alpha)}(x,y)$ employing integral forms and establish their characteristics, such as series definitions and generating functions. Also, we derive the higher-order truncated Tricomi functions and study their features.

Keywords truncated exponential polynomials, Laguerre polynomials, generalized Laguerre polynomials.
2010 AMS classification 33C45, 33C47.

1 Introduction

Truncated exponential polynomials have been shown to play an important role in evaluating integrals involving products of special functions in the physical sciences. These polynomials have several applications in physics, engineering and other research fields. Polynomials play a crucial role in applied mathematics as they can be characterized through various methods, including orthogonality criteria, generating functions, differential equations, integral transformations, recurrence relations and operational formulas. Mathematical and physical science researchers appreciate the useful properties of generalizations and extensions in their applications, making them an important tool in applied mathematics and approximation theory, where the capacity to alter functions in a variety of ways is frequently required. A handful of these polynomials and new polynomials related to them have recently been generated and analyzed, along with their properties, by a number of scientists who are studying special functions. (See for example [2, 3, 4, 5, 6, 7, 11, 14, 15, 16, 17, 19, 20, 21, 22, 23]).
The following exponential finite series determines the truncated exponential polynomials TEP $e_n(x)$ [1]:

$$e_n(x)=\sum_{k=0}^{n}\frac{x^k}{k!}, \tag{1}$$

which is the first sum of $(n+1)$ terms of the Maclaurin's series of e^x. Numerous optical and quantum mechanical issues contain these polynomials. In 2003, Dattoli *et al.* [12] provided the first comprehensive analysis of the characteristics of these polynomials. They admit the following integral representation [12]:

$$e_n(x)=\frac{1}{n!}\int_0^{+\infty}e^{-\zeta}\,(x+\zeta)^n d\zeta. \tag{2}$$

For truncated exponential polynomials, we have the following formula as generating function [12]:

$$\frac{1}{(1-t)}e^{xt}=\sum_{n=0}^{\infty}e_n(x)t^n.$$

It is worth to mention [1]:

$$\int_0^{+\infty}x^n e^{-x}dx=n!=\Gamma(n+1),\qquad n>0 \tag{3}$$

and

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$$\frac{1}{(1-t)^{\alpha}}=\sum_{k=0}^{+\infty}\frac{(\alpha)_k t^k}{k!}=\sum_{k=0}^{+\infty}\begin{bmatrix}\alpha+k-1\\ k\end{bmatrix}t^k, \qquad |\,t\,|<1,\ \alpha\in\mathbb{C}. \tag{4}$$

The n^{th}-order Tricomi function is defined by means of the following generating function [8]:

$$e^{(t-xt^{-1})}=\sum_{n=-\infty}^{+\infty}C_n(x)t^n, \quad n\in\mathbb{Z},\ t\neq 0, |\,x\,|<\infty,$$

accompanied by a definition of a series

$$C_n(x)=\sum_{k=0}^{+\infty}\frac{(-1)^k x^k}{k!(n+k)!}, \qquad n\in\mathbb{N}\cup\{0\}.$$

For $n=0$, above equation, gives 0^{th} order Tricomi function $C_0(x)$ [8]:

$$C_0(x)=\sum_{k=0}^{+\infty}\frac{(-1)^k x^k}{(k!)^2}, \qquad -\infty<x<\infty, \tag{5}$$

where

$$C_0(xt)=exp(-\hat{D}_x^{-1}t)\{1\}$$

and [18]:

$$\hat{D}_x^{-1}f(x):=\int_0^x f(\zeta)d\zeta, \quad \text{with} \quad \hat{D}_x^{-1}\{1\}=x \quad \text{and} \quad \left(\hat{D}_x^{-1}\right)^k\{1\}=\frac{x^k}{k!}, \quad \text{for } k\in\mathbb{N}\cup\{0\}.$$

The series definition for the extension of 0^{th} order Tricomi Bessel function of $C_\nu(x)$ is given as [18]:

$$C_\nu(x)=\sum_{k=0}^{+\infty}\frac{(-1)^k x^k}{k!\Gamma(\nu+k+1)}, \qquad \text{for } \nu\in\mathbb{C}. \tag{6}$$

We were inspired by the fact that the truncated exponential polynomials $e_n(x)$ have applications in several mathematical and scientific domains. Also, we were inspired by the work of Dattoli and his co-authors [12] on the features of these polynomials as well as their characteristics and their relatives, which are found in numerous optical and quantum mechanics problems and in evaluating integrals involving products of special functions. In this paper, our focus is to introduce 2-variable truncated Tricomi functions $h_n(x,y)$ and study their associated functions as well as their higher order functions. Particularly in section 2, we introduce 2-variable truncated Tricomi functions $h_n(x,y)$ through integral form and establish their characteristics, such as series definition, generating function, and differential equation. Also, we investigate the 2-parameter 2-variable truncated Tricomi functions $h_{n,\nu}^{(\alpha)}(x,y)$ and study its features. In section 3, we introduce higher order of 2-variable truncated Tricomi functions and study their properties.

2 2-variable truncated-Tricomi functions

In this section, we introduce the 2-variable truncated-Tricomi functions and study their properties. Also, we derive higher order truncated-Tricomi functions and study their features.

In the context of the following integral form for the truncated-Tricomi function $h_n(x)$ [12]:

$$h_n(x)=\frac{1}{n!}\int_0^{+\infty}e^{-\zeta}L_n(x,\zeta)d\zeta. \tag{7}$$

We define the 2-variable truncated Tricomi function by means of the following integral form:

$$h_n(x,y)=\frac{1}{n!}\int_0^{+\infty}e^{-\zeta}L_n(x,y\zeta)d\zeta, \tag{8}$$

where $L_n(x,y)$ is the 2-variable Laguerre polynomials specified by series [9, 13]:

$$L_n(x,y)=\ n!\sum_{k=0}^{n}\frac{(-1)^k x^k y^{n-k}}{(k!)^2(n-k)!} \tag{9}$$

and generating function

$$C_0(xt)e^{yt}=\sum_{n=0}^{+\infty}L_n(x,y)\frac{t^n}{n!}.$$

Utilizing formulas (9) and (3) on the right aspect of aforementioned equation, we get the series definition of 2-variable truncated Tricomi function

$$h_n(x,y)=\sum_{k=0}^{n}\frac{(-1)^k x^k y^{n-k}}{(k!)^2}. \tag{10}$$

Now, we establish the generating function of the 2-variable truncated Tricomi function $h_n(x,y)$ in the form of the following theorem:

Theorem 2.1. *The 2-variable truncated Tricomi function satisfies the following generating relation:*

$$\sum_{n=0}^{+\infty}h_n(x,y)t^n=\frac{C_0(xt)}{1-yt}. \tag{11}$$

Proof. In view of equation (10), we have

$$\sum_{n=0}^{+\infty}h_n(x,y)t^n=\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{(-1)^k x^k y^{n-k}}{(k!)^2}t^n,$$

which on employing the appropriate series rearrangement method [1]:

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}A(m,n)=\sum_{n=0}^{\infty}\sum_{m=0}^{n}A(m,n-m), \tag{12}$$

gives

$$\begin{aligned}\sum_{n=0}^{+\infty}h_n(x,y)t^n&=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-1)^k x^k y^{n}}{(k!)^2}t^{n+k}\\&=\sum_{n=0}^{\infty}y^n t^n\sum_{k=0}^{\infty}\frac{(-1)^k x^k}{(k!)^2}t^k.\end{aligned} \tag{13}$$

Using equations (4) and (5), gives assertion (11). □

Based on the integral form of 2-parameter truncated Tricomi function $h_{n,\nu}^{(\alpha)}(x)$ [12], we introduce the following integral form of 2-parameter 2-variable truncated Tricomi function $h_{n,\nu}^{(\alpha)}(x,y)$:

$$h_{n,\nu}^{(\alpha)}(x,y)=\frac{1}{\Gamma(n+\nu+1)}\int_0^{+\infty}e^{-\zeta}\zeta^{\alpha}L_n^{\nu}(x,y\zeta)d\zeta, \tag{14}$$

where $L_n^{\nu}(x,y)$ is the associated 2-variable Laguerre polynomials defined by series [10, 12]:

$$L_n^{\nu}(x,y)=\Gamma(n+\nu+1)\sum_{k=0}^{n}\frac{(-1)^k x^k y^{n-k}}{k!\ \Gamma(k+\nu+1)\ (n-k)!}. \tag{15}$$

After, using formulas (15) and (3), we obtain the series definition of 2-parameter 2-variable truncated Tricomi function $h_{n,\nu}^{(\alpha)}(x,y)$:

$$h_{n,\nu}^{(\alpha)}(x,y)=\sum_{k=0}^{n}\frac{(-1)^k x^k y^{n-k}\Gamma(n-k+\alpha+1)}{k!\ \Gamma(k+\nu+1)(n-k)!}. \tag{16}$$

The next theorem is used to define the generating function of 2-parameter 2-variable truncated Tricomi function $h_{n,\nu}^{(\alpha)}(x,y)$:

Theorem 2.2. *The generating function of 2-parameter 2-variable truncated Tricomi function $h_{n,\nu}^{(\alpha)}(x,y)$ can be expressed by the following formula:*

$$\sum_{n=0}^{+\infty}h_{n,\nu}^{(\alpha)}(x,y)t^n=\frac{\Gamma(\alpha+1)}{(1-yt)^{\alpha+1}}C_\nu(xt), \tag{17}$$

where $C_\nu(xt)$ is the associated Tricomi function, which defined in equation (6).

Proof. In view of equation (16), we have

$$\sum_{n=0}^{+\infty}h_{n,\nu}^{(\alpha)}(x,y)t^n=\sum_{n=0}^{+\infty}\sum_{k=0}^{n}\frac{(-1)^k x^k y^{n-k}\Gamma(n-k+\alpha+1)}{k!\Gamma(k+\nu+1)(n-k)!}t^n,$$

which on using equation (12), gives

$$\sum_{n=0}^{+\infty} h_{n,\nu}^{(\alpha)}(x,y)t^n = \sum_{n=0}^{+\infty}\sum_{k=0}^{+\infty} \frac{(-1)^k\Gamma(n+\alpha+1)x^k y^n}{k!\Gamma(k+\nu+1)n!} t^{n+k}$$
$$= \Gamma(\alpha+1)\sum_{n=0}^{+\infty} \begin{bmatrix} n+\alpha \\ n \end{bmatrix} y^n t^n \sum_{k=0}^{+\infty} \frac{(-1)^k x^k}{k!\Gamma(k+\nu+1)} t^k.$$

Simplification right hand side of aforementioned equation via formulas (4) and (6), we obtain assertion (17). □

Remark 1. For $y=1$ in equation (8), we get the integral form (7) for the truncated Tricomi function $h_n(x)$. For $y=1$ in equation (10) and (11), we get the series definition and generating function for the truncated Tricomi function $h_n(x)$ given in [12]. Also, for $y=1$ in equations (14), (16) and (17), we get the integral form, series definition and generating function for the 2-parameter truncated Tricomi function $h_{n,\nu}^{(\alpha)}(x)$ given in [12]. Further, for $\alpha=0$ in equations (14), (16) and (17), we get the following respective integral form, series definition and generating function of 2-variable truncated associated Tricomi function:

$$h_{n,\nu}(x,y) = \frac{1}{\Gamma(n+\nu+1)} \int_0^{+\infty} e^{-\zeta} L_n^{\nu}(x, y\zeta) d\zeta, \tag{18}$$

$$h_{n,\nu}(x,y) = \sum_{k=0}^{n} \frac{(-1)^k x^k y^{n-k}}{k!\Gamma(k+\nu+1)} \tag{19}$$

and

$$\sum_{n=0}^{+\infty} h_{n,\nu}(x,y)t^n = \frac{C_\nu(xt)}{1-yt}. \tag{20}$$

3 Higher-order 2-variable truncated Tricomi functions

In this section, we introduce the higher order of 2-variable truncated Tricomi function by using m^{th}-order Laguerre-type polynomials and study their properties.

Now, we define m^{th}-order 2-variable truncated Tricomi function by means of the following integral forms:

$${}_{[m]}h_n(x,y) = \frac{1}{n!} \int_0^{+\infty} e^{-\zeta} \ {}_{[m]}L_n(x, \zeta y) d\zeta, \tag{21}$$

where the notation [ ] indicated is a square bracket and ${}_{[m]}L_n(x,y)$ is the generalization of the 2-variable Laguerre polynomials specified by series [12]:

$${}_{[m]}L_n(x,y) = n! \sum_{k=0}^{[n/m]} \frac{(-x)^{n-mk} y^k}{((n-mk)!)^2 k!}, \tag{22}$$

with generating function

$$e^{yt^m} C_0(xt) = \sum_{n=0}^{+\infty} {}_{[m]}L_n(x,y) \frac{t^n}{n!}.$$

Using equation (22) in equation (21), then using equation (3) in the right hand side of the resultant equation and comparing the coefficients of the equal powers of t from both sides of that equation, we obtain the series definition of m^{th}-order truncated exponential polynomials ${}_{[m]}h_n(x,y)$:

$${}_{[m]}h_n(x,y) = \sum_{k=0}^{[n/m]} \frac{(-x)^{n-mk} y^k}{((n-mk)!)^2}. \tag{23}$$

The follwing theorem is used to define the generating function of m^{th}-order 2-variable truncated Tricomi function ${}_{[m]}h_n(x,y)$:

Theorem 3.1. *The m^{th} order 2-variable truncated Tricomi function ${}_{[m]}h_n(x,y)$ satisfy the following generating function:*

$$\sum_{n=0}^{+\infty} {}_{[m]}h_n(x,y)t^n = \frac{C_0(xt)}{1-yt^m}. \tag{24}$$

Proof. In view of equation (23), we have

$$\sum_{n=0}^{+\infty} {}_{[m]}h_n(x,y)t^n = \sum_{n=0}^{+\infty}\sum_{k=0}^{[n/m]} \frac{(-x)^{n-mk}y^k}{((n-mk)!)^2}t^n,$$

which on using the following series rearrangement technique[1]:

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} A(k,n) = \sum_{n=0}^{\infty}\sum_{k=0}^{[n/m]} A(k,n-mk), \tag{25}$$

gives

$$\begin{aligned}\sum_{n=0}^{+\infty} {}_{[m]}h_n(x,y)t^n &= \sum_{n=0}^{+\infty}\sum_{k=0}^{+\infty} \frac{(-x)^n y^k}{(n!)^2}t^{n+mk}\\ &= \sum_{n=0}^{+\infty} \frac{(-x)^n}{(n!)^2}t^n \sum_{k=0}^{+\infty} y^k t^{mk},\end{aligned}$$

which on using equations (4) and (5), gives assertion (24). □

Remark 2. Since, in view of equation (22), for $m=2$, the m^{th}-order 2-variable Laguerre polynomials ${}_{[m]}L_n(x,\zeta)$ reduced to the 2^{nd}-order 2-variable Laguerre polynomials ${}_{[2]}L_n(x,\zeta)$. Thus, for $m=2$ in equations (21), (23) and (24), the m^{th}-order 2-variable truncated Tricomi function reduces to the 2^{nd}-order 2-variable truncated Tricomi function as follows:

$${}_{[2]}h_n(x,y) = \frac{1}{n!}\int_0^{+\infty} e^{-\zeta}\ {}_{[2]}L_n(x,y\zeta)d\zeta, \tag{26}$$

$${}_{[2]}h_n(x,y) = \sum_{k=0}^{[n/2]} \frac{x^{n-2k}y^k}{((n-2k)!)^2}, \tag{27}$$

and

$$\sum_{n=0}^{+\infty} {}_{[2]}h_n(x,y)t^n = \frac{C_0(xt)}{1-yt^2}, \tag{28}$$

respectively.

To introduce the 2-parameter m^{th}-order 2-variable truncated Tricomi function, we need to introduce the associated m^{th}-order 2-variable Laguerre polynomials ${}_{[m]}L_n^{\nu}(x,y)$ through the following generating function:

$$C_\nu(xt^m)e^{yt} = \sum_{n=0}^{+\infty} {}_{[m]}L_n^{\nu}(x,y)\frac{t^n}{\Gamma(n+\nu+1)}. \tag{29}$$

Expanding the aforementioned equation by using equation (6) then comparing the equal powers of t from both sides of the resultant equation, we obtain

$${}_{[m]}L_n^{\nu}(x,y) = \Gamma(n+\nu+1)\sum_{k=0}^{[n/m]} \frac{(-x)^k y^{n-mk}}{k!\Gamma(k+\nu+1)(n-mk)!}. \tag{30}$$

We are now defining the 2-parameter m^{th}-order truncated Tricomi function in two variables ${}_{[m]}h_{n,\nu}^{(\alpha)}(x,y)$ by means of the following integral form:

$${}_{[m]}h_{n,\nu}^{(\alpha)}(x,y) = \frac{1}{\Gamma(n+\nu+1)}\int_0^{+\infty} e^{-\zeta}\zeta^{\alpha}\ {}_{[m]}L_n^{\nu}(x,\zeta y)d\zeta, \tag{31}$$

which on using equations (30) and (3), we obtain the following series definition of 2-parameter m^{th}-order truncated Tricomi function ${}_{[m]}h_{n,\nu}^{(\alpha)}(x,y)$:

$${}_{[m]}h_{n,\nu}^{(\alpha)}(x,y) = \sum_{k=0}^{[n/m]} \frac{(-x)^k y^{n-mk}\Gamma(n-mk+\alpha+1)}{k!\Gamma(k+\nu+1)(n-mk)!}. \tag{32}$$

Next, by demonstrating the subsequent theorem, we obtain the generating function for the 2-parameter m^{th}-order 2-variable truncated Tricomi function ${}_{[m]}h_{n,\nu}^{(\alpha)}(x,y)$.

Theorem 3.2. *The 2-parameter m^{th}-order truncated Tricomi function possesses the following generating function:*

$$\sum_{n=0}^{+\infty} {}_{[m]}h_{n,\nu}^{(\alpha)}(x,y)t^n = \frac{\Gamma(\alpha+1)}{(1-yt)^{\alpha+1}}C_\nu(xt^m), \tag{33}$$

where $C_\nu(xt^m)$ is the associated m^{th}-order Tricomi function defined by equation (6).

Proof. In the context of equation (32), we have

$$\sum_{n=0}^{+\infty} {}_{[m]}h_{n,\nu}^{(\alpha)}(x,y)t^n = \sum_{n=0}^{+\infty}\sum_{k=0}^{[n/m]} \frac{\Gamma(n-mk+\alpha+1)(-x)^k y^{n-mk}}{k!\Gamma(k+\nu+1)(n-mk)!} t^n,$$

which on using equation (25), gives

$$\begin{aligned}\sum_{n=0}^{+\infty} {}_{[m]}h_{n,\nu,q}^{(\alpha)}(x,y)t^n &= \sum_{n=0}^{+\infty}\sum_{k=0}^{+\infty} \frac{\Gamma(n+\alpha+1)(-x)^k y^n}{k!\Gamma_q(k+\nu+1)n!} t^{n+mk} \\ &= \Gamma(\alpha+1)\sum_{n=0}^{+\infty}\begin{bmatrix} n+\alpha \\ n\end{bmatrix} y^n t^n \sum_{k=0}^{+\infty}\frac{(-x)^k}{k!\Gamma(k+\nu+1)} t^{mk}. \end{aligned} \tag{34}$$

Additionally, by plugging equations (4) and (6), into the right part of equation (34), gives assertion (33). □

Remark 3. The 1-parameter m^{th}-order 2-variable truncated Tricomi function can be defined by means of the following integral form, series definition and generating function:

$${}_{[m]}h_{n,\nu}(x,y) = \frac{1}{\Gamma(n+\nu+1)}\int_0^{+\infty} e^{-\zeta}\ {}_{[m]}L_n^{\nu}(x,y\zeta)d\zeta, \tag{35}$$

$${}_{[m]}h_{n,\nu}(x,y) = \sum_{k=0}^{[n/m]} \frac{(-x)^k y^{n-mk}}{k!\Gamma(k+\nu+1)} \tag{36}$$

and

$$\sum_{n=0}^{+\infty} {}_{[m]}h_{n,\nu}(x,y)t^n = \frac{C_\nu(xt^m)}{1-yt}, \tag{37}$$

by taking $\alpha=0$ in equations (31), (32) and (33), respectively.

Likewise, we obtain the integral form, series definition, and generating function of the 1-parameter 2^{nd}-order 2-variable truncated Tricomi function by taking $m=2$ in equations (35), (36), and (37):

$${}_{[2]}h_{n,\nu}(x,y) = \frac{1}{\Gamma(n+\nu+1)}\int_0^{+\infty} e^{-\zeta}\ {}_{[2]}L_n^{\nu}(x,y\zeta)d\zeta, \tag{38}$$

$${}_{[2]}h_{n,\nu}(x,y) = \sum_{k=0}^{[n/2]} \frac{(-x)^k y^{n-2k}}{k!\Gamma(k+\nu+1)} \tag{39}$$

and

$$\sum_{n=0}^{+\infty} {}_{[2]}h_{n,\nu}(x,y)t^n = \frac{C_\nu(xt^2)}{1-yt}, \tag{40}$$

respectively.

Finally, we obtain the integral form, series definition, and generating function for the 2-parameter truncated q-Tricomi function ${}_{[m]}h_{n,\nu}^{(\alpha)}(x)$ by taking $y=1$ in equations (31), (32), and (33).

$${}_{[m]}h_{n,\nu}^{(\alpha)}(x) = \frac{1}{\Gamma(n+\nu+1)}\int_0^{+\infty} \zeta^\alpha exp(-\zeta)\ {}_{[m]}L_n^{\nu}(x,\zeta)d\zeta, \tag{41}$$

$${}_{[m]}h_{n,\nu}^{(\alpha)}(x) = \sum_{k=0}^{[n/m]} \frac{x^k\Gamma(n-mk+\alpha+1)}{k!\Gamma(k+\nu+1)(n-mk)!}, \tag{42}$$

and

$$\sum_{n=0}^{+\infty} {}_{[m]}h_{n,\nu}^{(\alpha)}(x)t^n = \frac{C_\nu(xt^m)}{(1-t)^{\alpha+1}}, \tag{43}$$

respectively.

4 Conclusion

Recently, many special functions have been used to study calculus. Amazingly, this led to the discovery of the truncated exponential polynomials and the Laguerre polynomials and the study of their properties. In this study, the 2-variable truncated Tricomi function and its properties have been examined using the Euler representation of the gamma function. In particular, the 2-variable truncated Tricomi function $h_n(x,y)$, 2-parameter 2-variable truncated $h_{n,\nu}^{(\alpha)}(x,y)$ and the m^{th}-order 2-variable truncated Tricomi function ${}_{[m]}h_n(x,y)$ have been introduced utilizing the integral forms and studied their characteristics, such as series definitions and generating functions. In addition, we have introduced the associated m^{th}-order 2-variable q-Laguerre polynomials ${}_{[m]}L_n^{\nu}(x,y)$ and we have derive the 2-parameter m^{th}-order truncated Tricomi function ${}_{[m]}h_{n,\nu}^{(\alpha)}(x,y)$ and studied their features. We want to learn more about the new special functions and polynomials and find out how they can be used in mathematics, science, as well as engineering.

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