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Non-perturbative Renormalization of the Complete Basis of Four-fermion Operators and B-parameters

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We present results on the B-parameters B_K, $B_7^{3/2}$ and $B_8^{3/2}$, at $\beta = 6.0$, with the tree-level Clover action. The renormalization of the complete basis of dimension-six four-fermion operators has been performed non-perturbatively. Our results for B_K and $B_7^{3/2}$ are in reasonable agreement with those obtained with the (unimproved) Wilson action. This is not the case for $B_8^{3/2}$. We also discuss some subtleties arising from a recently proposed modified definition of the B-parameters.

1. Operator Renormalization

In the present talk, we are interested in the determination of three B-parameters, namely B_K, $B_7^{3/2}$ and $B_8^{3/2}$. B_K measures the deviation of the $\Delta S = 2$ matrix element $\langle K^0|O_{\Delta S=2}|\bar{K}^0\rangle$ from its value in the Vacuum Saturation Approximation (VSA). $B_7^{3/2}$ and $B_8^{3/2}$ measure the deviation of the $\Delta I = 3/2$ matrix elements $\langle \pi|O_{7,8}^{3/2}|\bar{K}\rangle$ from their VSA values. We recall that B_K is an essential ingredient to the determination of the CP-violation parameter ϵ, whereas $B_7^{3/2}$ and $B_8^{3/2}$ are needed in the determination of the ratio ϵ'/ϵ. All three matrix elements can be computed on the lattice from three-point correlation functions, involving the so-called "eight" diagrams. Their renormalization has no power subtraction (involving "eye" diagrams).

The main novelty of the present work, which is an extension of [1], is the implementation of the Non-Perturbative Method (NPM) for the renormalization of the corresponding operators. We have determined the operator mixing for the complete basis of four-fermion operators with the aid of the discrete symmetries (parity, charge conjugation and switching of flavours). For the parity-conserving operators, relevant to this work, we have used the following complete basis of five operators:

$$\begin{aligned} Q_{1,2} &= V \times V \pm A \times A, \\ Q_{3,4} &= S \times S \mp P \times P, \\ Q_5 &= T \times T. \end{aligned} \tag{1}$$

In these expressions, $\Gamma \times \Gamma$ (with $\Gamma = V, A, S, P, T$ a generic Dirac matrix) stands for $\frac{1}{2}(\bar{\psi}_1\Gamma\psi_2\bar{\psi}_3\Gamma\psi_4 + \bar{\psi}_1\Gamma\psi_4\bar{\psi}_3\Gamma\psi_2)$, where ψ_i, $i = 1, \ldots, 4$ are fermion fields with flavours chosen so as to reproduce the desired operators (see [2,3] for details): the parity-conserving component of the four-fermion operator $O_{\Delta S=2}$ corresponds to Q_1 in our basis, whereas the parity-conserving parts of $O_7^{3/2}$ and $O_8^{3/2}$ are (up to numerical factors) Q_2 and Q_3. On the lattice, these operators mix under renormalization in the following pattern

$$\begin{aligned} \hat{Q}_1 &= Z_{11}Q_1^s, \\ \hat{Q}_2 &= Z_{22}Q_2^s + Z_{23}Q_3^s, \\ \hat{Q}_3 &= Z_{32}Q_2^s + Z_{33}Q_3^s. \end{aligned} \tag{2}$$

Z_{11} and Z_{ij} (with $i,j = 2,3$) are logarithmically divergent renormalization constants which depend on the coupling and $a\mu$. These are renor-

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malizations which occur also in the continuum. The subtractions

$$Q_1^s = Q_1 + \sum_{i=2}^{5} Z_{1i} Q_i \ , \quad (3)$$
$$Q_i^s = Q_i + \sum_{j=1,4,5} Z_{ij} Q_j \quad (i=2,3) \ ,$$

occur on the lattice because of the chiral symmetry breaking Wilson term in the action. The mixing coefficients in the last expressions are finite and only depend on the lattice coupling $g_0^2(a)$. The results for all the renormalization constants Z_{ij} (computed with the NPM at several renormalization scales μ at $\beta = 6.0$) can be found in [2].

The finite mixing coefficients have also been determined in [4], using Ward Identities (WI). The NPM and WI determinations are equivalent at large enough scale μ; see [2,5] for explicit demonstrations. It is not true, as claimed in [4] that the WI method is theoretically more sound, off the chiral limit. On the other hand, we have checked that the choice of operator basis made in [4] appears to give stabler results in practice. This is not, however, a question of principle.

2. The Definition of B_K

The standard definition of B_K is given by

$$B_K(\mu) = \frac{\langle \bar{K}^0 | \hat{O}_{\Delta S=2}(\mu) | K^0 \rangle}{\langle \bar{K}^0 | O_{\Delta S=2} | K^0 \rangle_{VSA}} = \frac{\langle \bar{K}^0 | \hat{O}_{\Delta S=2}(\mu) | K^0 \rangle}{\frac{8}{3} f_K^2 m_K^2} \quad (4)$$

Note that the operator $\hat{O}_{\Delta S=2}$ in the numerator is renormalized. Thus the numerator of the above ratio is a μ-dependent quantity, whereas the denominator is a physical one. Thus defined, B_K scales with μ in the same way as $\hat{O}_{\Delta S=2}$. In [4], the following modified definition has also been used:

$$B'_K(\mu) = \frac{\langle \bar{K}^0 | \hat{O}_{\Delta S=2}(\mu) | K^0 \rangle}{\langle \bar{K}^0 | \hat{O}_{\Delta S=2}(\mu) | K^0 \rangle_{VSA}} = \frac{\langle \bar{K}^0 | O^s_{\Delta S=2}(a) | K^0 \rangle}{\langle \bar{K}^0 | O^s_{\Delta S=2}(a) | K^0 \rangle_{VSA}} \quad (5)$$

In other words, each operator subtraction in the renormalization of $\hat{O}_{\Delta S=2}(\mu)$ is vacuum-saturated in the denominator. This modified definition results in a statistically stabler signal. In [4], both B_K and B'_K were measured at several β values and the results, after being extrapolated at zero lattice spacing, were found to be compatible. However, it can be shown [6] that this definition has serious shortcomings. The problem lies with the denominator, which, up to terms proportional to the lattice spacing, behaves like

$$\frac{1}{Z_A^2} \frac{8}{3} f_K^2 m_K^2 + Z_P(g_0^2) |\langle 0 | P(a) | K^0 \rangle|^2 \quad (6)$$

where Z_A and $Z_P(g_0^2)$ are the renormalization constants of the axial current A_μ and the pseudoscalar density P. The numerator has the correct chiral behaviour, but that of the denominator is spoiled by $\mathcal{O}(g_0^2)$ terms. These terms could be eliminated by extrapolating to the continuum limit $a \to 0$ before taking the chiral limit (in the continuum limit the denominator of B'_K reduces to that of B_K). But this is not possible, as the numerator diverges in this limit. A possible remedy of this problem would be a further modification of the definition of B_K:

$$B_K(\mu)'' = \frac{Z_O(a\mu)}{Z_A^2} B'_K(\mu) \quad (7)$$

which has a finite numerator (also in the continuum limit) and a denominator

$$\frac{8}{3} f_K^2 m_K^2 + \frac{Z_P}{Z_A^2}(g_0^2) |\langle 0 | P(a) | K^0 \rangle|^2 \quad (8)$$

The last term scales like $[g_0^2]^{3/11}$, since $Z_P(g_0^2) \sim g_0^2$ and $P(a) \sim [g_0^2]^{-8/11}$. Thus it vanishes very slowly in the continuum limit, and cannot be removed by a linear extrapolation in a (as suggested in [4]).

3. Results

Our results have been obtained with the tree-level Clover action, at $\beta = 6.0$ in the quenched approximation. The matrix elements have been computed on an $18^3 \times 64$ lattice (460 configurations), whereas the non-perturbative renormalization (based on the computation of the matrix

Table 1
B-parameters for $\Delta S = 2$ and $\Delta I = 3/2$ operators at the renormalization scale $\mu = a^{-1} \approx 2$ GeV. All results are in the $\overline{\text{MS}}$ renormalization scheme (with the dimensional regualrization shown in the third column). $\hat{B}_K$ is the RGI B-parameter (obtained by multiplying B_K by its Wilson coefficient).

	NPM	NDR	0.66(11)	this work
B_K	BPT	NDR	0.65(11)	this work
	BPT $q^* = 1/a$	NDR	0.74(4)	[7]
	NPM	NDR	0.93(16)	this work
$\hat{B}_K$	BPT	NDR	0.92(16)	this work
	NPM	NDR	0.72(5)	this work
$B_7^{3/2}$	BPT	DRED	0.65(2)	this work
	BPT $q^* = 1/a$	NDR	0.58(2)	[7]
	BPT $q^* = \pi/a$	NDR	0.65(2)	[7]
	NPM	NDR	1.03(3)	this work
$B_8^{3/2}$	BPT	DRED	0.71(2)	this work
	BPT $q^* = 1/a$	NDR	0.81(3)	[7]
	BPT $q^* = \pi/a$	NDR	0.84(3)	[7]

elements of the operators between quark states) has been performed on a $16^3 \times 32$ lattice (100 configurations). In table 1 we present our results and compare them to those of [7], also obtained at $\beta = 6.0$, but with the (unimproved) Wilson action and with the operator renormalization done in Boosted Perturbation Theory (BPT), which involves an "optimal" renormalization scale q^*. We also show our preliminary analysis in BPT, for comparison (our BPT prescription does not make use of q^*; see [3] for details). Any differences arising from the use of two regularization schemes (NDR and DRED) in $\overline{\text{MS}}$ are small and are properly accounted for in [3].

Our results for B_K, obtained with the NP and the BPT renormalization of the operators are in perfect agreement. With a larger statistical error, our B_K value also agrees with those of [7]. Also for $B_7^{3/2}$, our NPM and BPT values are in good agreement and fully compatible with the results of [7] (for large enough q^*). We find, instead, a large difference between our NPM and BPT estimates of $B_8^{3/2}$. Our value obtained with BPT is close to that of [7], where the Wilson action was used. The NPM estimate, instead, is in disagreement with any value obtained in BPT (either with the Wilson or the Clover action and for several boosting variants). We believe that the difference between our NPM estimate and that of [7] is due to the NPM used in the former result, rather than the implementation of different actions (Clover and Wilson respectively). The increase in the NPM value of $B_8^{3/2}$ is of great phenomenological interest, since it may induce a considerable decrease of the ratio ϵ'/ϵ.

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