Chapter 33
Fractal Neurodynamics

Karolina Armonaite, Livio Conti, and Franca Tecchio

Abstract The neuronal ongoing electrical activity in the brain network, the neurodynamics, reflects the structure and functionality of generating neuronal pools. The activity of neurons due to their excitatory and inhibitory projections is associated with specific brain functions. Here, the purpose was to investigate if the local ongoing electrical activity exhibits its characteristic spectral and fractal features in wakefulness and sleep across and within subjects. Moreover, we aimed to show that measures typical of complex systems catch physiological features missed by linear spectral analyses. For this study, we concentrated on the evaluation of the power spectral density (PSD) and Higuchi fractal dimension (HFD) measures. Relevant clinical impact of the specific features of neurodynamics identification stands primarily in the potential of classifying cortical parcels according to their neurodynamics as well as enhancing the effectiveness of neuromodulation interventions to cure symptoms secondary to neuronal activity unbalances.

Keywords Neurodynamics · Power spectral density · Power law · Higuchi fractal dimension · Local signature

33.1 EEG-Derived Neurodynamics Assessment in the Human Brain

The statistical properties of electrical signals produced in the brain are of great interest as they contain a harvest of information that is not yet fully understood, show specificity of different source areas and exhibit unpredictability due to their

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irregular behaviours and non-stationarity of the underlying generating mechanisms. These processes characterize the complex internal brain activities when neuronal assemblies are synchronously coupled and switch between the states during brain functioning instead of external stimuli [14]. Other researchers prefer to regard the brain as a dynamical system, since majority of neuronal assemblies are excitable non-linear sub-systems possessing self-organization and power-law scaling properties [4, 10, 31, 35].

There are several claims that the brain's electrical ongoing activity is deemed to be scale-free if its power spectrum follows power law distribution $P(f) \sim 1/f^{\beta}$ [19, 31, 58], which is also a prerequisite for fractality [28]. If the brain oscillations have been well studied up to date, the non-oscillatory components are still a topic for discussion, and the methods for interpretation as well as the robust estimation of the scale-free neurodynamics are still being studied [20].

Nevertheless, evidence suggests that fractal properties of neurodynamics are handy to evaluate the complexity of implied processes and can have a direct application in clinics. More precisely, the fractal measures have been useful to distinguish consciousness from unconscious states, e.g. sleep stages and depth of anaesthesia [6, 18, 23, 49], fractal signatures can be also used as a biomarker in detecting senile dementia [5, 51] and are claimed to be a better predictor of schizophrenia than neural spectral oscillations [45]. In addition, fractal properties of EEG on the scalp and in intracranial recordings have been investigated to seek a signature of different cortical areas [7, 8, 17].

In this chapter, we will attempt to describe the local cortical neurodynamics via spectral and fractal analyses of electrophysiological time series and discuss their limits. The aim is to highlight the existence of typical features in distinct cortical parcels and that these properties are maintained even during sleep/wake stages. Moreover, cited studies confirmed the hypothesis that the measures of system complexity can better detect the specificities of the ongoing electrical activity than the linear ones such as the spectral analysis.

33.2 Neurodynamics as Local Cortical Signature: A Spectral Estimate

The first classification of the brain started with Korbinian Brodmann, who suggested the anatomical parcelling of the cortex based on the cytology of the constituent neurons [9]. The proof of different brain structural properties raised the notion to study their function separately and the functional connectivity between them [54].

While great interest has been given to studying the task-related behaviours, up to date, the possible brain parcellation based on intrinsic characteristics of the neuroelectric activity during resting states was barely investigated. The attempt to find a 'fingerprint' of distinct brain areas in EEG recordings was suggested by clustering them on the basis of distinct spectral profiles of their electrical activity.

Keitel and Gross [36] demonstrated that each brain area engages in different spectral behaviour that is characteristic of individual areas.

Cottone et al. [17] deployed the power spectrum measure to identify the local neurodynamics of the primary motor cortex (M1) and primary somatosensory cortex (S1) from EEG recordings on the scalp of 20 subjects, in both hemispheres, during resting wakefulness with eyes open and eyes closed, sensory stimulation and performing simple motor functions. The authors have shown that the neurodynamics' spectral indices displayed a clear and different pattern for S1 and M1 areas. They did not notice a distinction between the left and right side; however, all subjects were showing clear alpha (8–12 Hz) frequency band activity in S1 and high beta (26–33 Hz) and gamma (33–80 Hz) activity in M1.

More recently, the investigation of spectral properties of different brain regions was published by Montreal Neurological Institute (MNI) using stereotactic EEG recordings [25] demonstrating that different brain areas in resting wakefulness possess a characteristic activity. The authors have shown that alpha rhythm is found in the occipital lobe, the parietal lobe and temporal lobes. Beta rhythm is frequently found in the anterior head regions. In particular, well-sustained beta frequencies are described in the precentral and postcentral gyri with peaks at lower frequencies present in the postcentral regions. The precentral gyrus also expresses gamma activity. Moreover, beta peaks are found in the middle cingulate gyrus and anterior insula [25].

Following Cottone et al. [17] and Frauscher et al. [25] results, another attempt to analyse the typical spectral features was made by Armonaite et al. [7]. The authors studied three primary cortices: auditory (A1), somatosensory (S1) and motor (M1) from MNI intracranial stereotactic EEG recordings (sEEG). The distribution of the electrode contacts across three brain parcels of interest is shown in Fig. 33.1 where a single channel per subject per area has been selected (representative channel). The authors analysed the local neurodynamics characteristic spectral features, initially, within a subject and later across the population.

33.2.1 Spectral Features in Resting Wakefulness

The purpose of the Armonaite et al. [7] study was to deepen knowledge of the brain neurodynamics as cortical area signature. For extracting the spectral properties of the signals, authors calculated the power spectral density (PSD) of each channel in the three investigated regions during resting wakefulness (Fig. 33.2). The spectrum was categorized into 7 frequency bands: delta ($\leq$4 Hz), theta (4–8 Hz), alpha (8–12 Hz), low beta (12–26 Hz), high beta (26–33 Hz), low gamma (33–49 Hz) and high gamma (49-80 Hz).

Initially, the authors compared the PSD in the three regions of interest within a subject and then averaged across the population. In Fig. 33.3, the scatterplots of the PSD evaluated in each frequency band, for each cortical region separately in each subject, are compared between a couple of areas (M1 vs S1, M1 vs A1

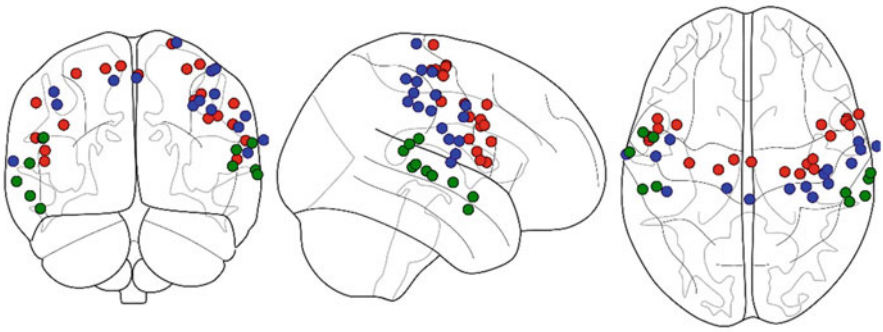

Fig. 33.1 The distribution of representative channels in A1, S1 and M1. After Armonaite et al. [7]. After a representative channel selection for each subject, the number of subjects that followed the constrain of the Armonaite et al. [7] study was reduced to 10 subjects in A1 (green), in S1 (blue) to 17 subjects and in M1 (red) to 20 subjects. The location of the selected representative channel for each subject in each area is shown on the cortex image in three projections

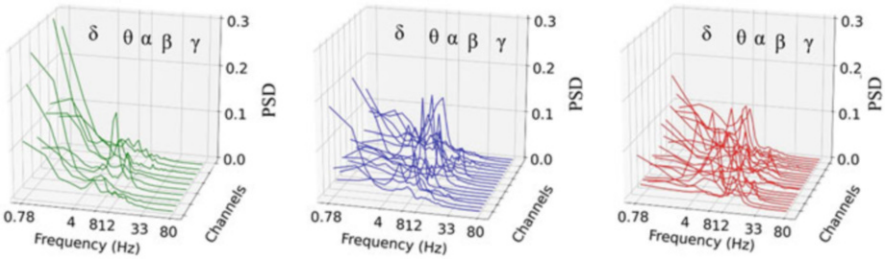

Fig. 33.2 The PSD for each channel in A1, S1 and M1. After Armonaite et al. [7]. The normalized PSD was evaluated for each channel in each region. To spot the trend at lower frequencies, the x-axis is represented on the logarithmic scale. From the left: A1, (10 channels) PSD expresses the highest power in the delta band; S1, (17 channels) PSD exhibited higher power in theta and alpha; M1, (20 channels) PSD expressed high power in theta, alpha, beta and gamma frequency bands

and S1 vs A1) in the wake. Only scatterplots for the frequency bands where the two sources statistically differ are presented. The bisector of the rectangular plot areas, divides the upper and lower regions where the PSD of the compared cortical parcels dominates. It is evident that the local neurodynamics evaluated in each subject appeared specific for the cortical district in terms of PSD band values. S1 consistently shows greater power than M1, evaluated with Wilcoxon nonparametric test (W_{test}), in alpha band [$W_{\text{test}} = 26, p = 0.03$] and smaller in high beta [$W_{\text{test}} = 25$, $p = 0.03$], low gamma [$W_{\text{test}} = 15$, $p = 0.01$] and high gamma [$W_{\text{test}} = 28$, $p = 0.04$]. A1 has power in delta frequency band greater than M1 (9 out of 9 subjects) while smaller in low beta (7/9), high beta (7/9) and low gamma (7/9) bands. A1 steadily shows greater power than S1 in delta band (6/6) and smaller in low (5/6) and high beta (4/6) bands.

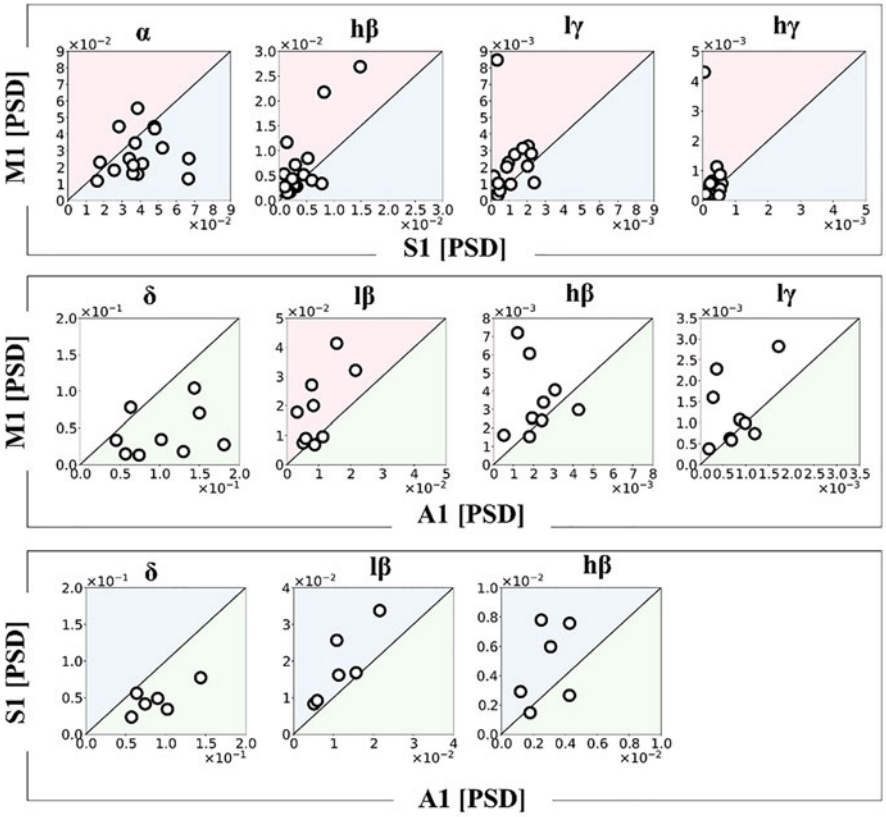

Fig. 33.3 Local neurodynamics comparisons in frequency bands. After Armonaite et al. [7]. Comparing the neurodynamics between couples of the studied brain areas, the scatterplots of the normalized PSD for each subject are shown for the frequency bands where the PSD of two sources statistically differ. **Top panel**: M1 versus S1 [16 subjects] in 4 bands. **Middle panel**: M1 versus A1 [9 subjects] in 4 bands. **Bottom panel**: S1 versus A1 [6 subjects] in 3 bands. In each plot, a point above (below) the diagonal has a PSD of the source represented on x-axis lower (higher) than that of the source shown on y-axis. The Greek letter stands for the frequency bands, where the l/h prefix indicates low/high range, respectively

Further, the PSD – averaged across all channels – shows specific features between cortical areas also at the population level. The PSD in delta band is prevalent for A1, in alpha for S1 and in high beta and gamma for M1 (Fig. 33.4).

33.2.2 Spectral Features During Sleep

There is a large amount of literature suggesting that the statistical features of the brain activity measured via electrophysiological tools can vary across sleep

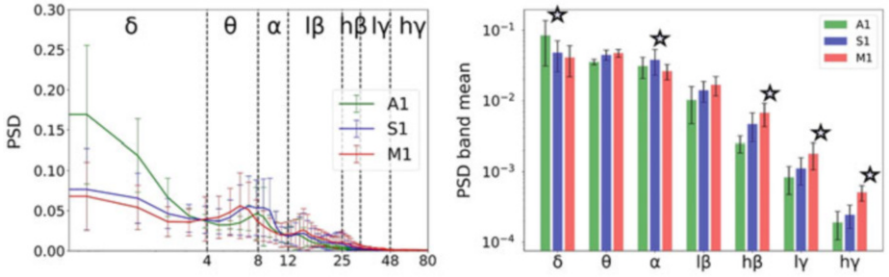

Fig. 33.4 PSD across population. Modified from Armonaite et al. [7]. In the left panel, the normalized PSD averaged across representative channels of each subject is given for A1 (green), S1 (blue) and M1 (red). In the right panel, the bars indicate PSD mean on each frequency band, always averaged over representative channels, where the error bar is the standard deviation of the frequency band for all 7 frequency bands. The stars indicate those bands where the highest PSD has been found across regions, i.e. δ prevailing in A1; α in S1 and from hβ in M1. To enhance the higher frequencies, we show y-axis on logarithmic scale

stages [11, 15, 18, 44]; however, there is a lack of evidence that the differences of neurodynamics of separate brain areas can maintain between in the diverse sleep stages.

The variations of oscillatory frequency components were well described by Von Ellenrieder et al. [52] across regions during sleep from sEEG data. They found less heterogeneity of different regions in N2 and N3 sleep stages with respect to the variability of frequency components across different brain regions during wakefulness and rapid eye movement (REM) sleep stage. They also observed that, across different sleep stages, the clear peaks were prevalent more in the limbic system and in mesial visual and motor cortices, whereas more homogeneity was found in the rest of the investigated regions.

Armonaite et al. [8] further studied the neurodynamics in three sleep stages (N2, N3 and REM) of primary motor, somatosensory and auditory cortices, by measuring their properties via power spectral densities, in order to assess if the local neurodynamics of investigated cortical regions maintains its' spectral features and differentiation not only in wake, but also across different sleep stages both within a subject and across population.

It was observed that within a subject in REM sleep M1 had greater power than S1 in low beta, high beta and low gamma bands [$W_{\text{test}} = 11$, $p = 0.05$, $W_{\text{test}} = 4$, $p = 0.01$, $W_{\text{test}} = 5$, $p = 0.01$], respectively. It was also found that M1 had greater PSD than S1 in alpha band [$W_{\text{test}} = 24$, $p = 0.07$] in N2 sleep stage. Likewise, also in N3 stage alpha waves were prevailing more in M1 than in S1 with [$W_{\text{test}} = 18$, $p = 0.03$] [8]. However, across population, systematic cortical area differences were not observed unlike in wakefulness (Fig. 33.5).

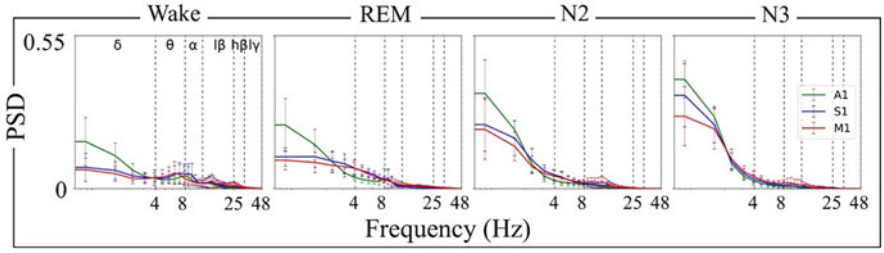

Fig. 33.5 PSD averaged across population in three regions of interest, in four sleep stages. After Armonaite et al. [8]. The mean and standard deviation of normalized PSD as a function of frequency calculated across all subjects, separately for M1, S1 and A1 areas, are presented. PSD values are normalized so that the area under each curve is equal to 1

33.3 Fractal Neurodynamics: Properties and Estimate Methods

As the large literature on the subject demonstrates, the brain's electromagnetic signals show complex properties and are still a topic for intense discussion, as they exhibit irregular behaviours that the spectral analysis might not be effective to catch. In fact, the electrophysiological signals contain not only harmonic oscillations but also noisy fluctuations that might be as informative [60]. These components are being suggested to be approached as scale-free and even fractal properties [28].

Some authors are quite sceptical about the scale-free, self-organized and fractal nature of brain neurodynamics, claiming that these noisy fluctuations occur merely due to the simple superposition of random components acting on multiple time scales [30]. Other researchers support also the idea that the observed power law of the power spectrum, so-called $1/f^{\beta}$ noise, would be an artefact due to many averaged narrow-band periodic oscillations of different amplitudes and frequencies [13], or could be due to the rapid exponential rise and slow exponential decay of dendritic response to an impulse input, possibly convolved with Poisson process pulses [26]. Moreover, Evertz et al., in a recent paper suggested a model according to the $1/f^{\beta}$ property would be the result of several independent perturbed damped alpha oscillations [22]. It was demonstrated that oscillatory alpha power correlates with the value of the β power law exponent of PSD at high frequencies, therefore, subjects with higher rhythmical alpha power would express higher β exponent for high frequencies [42] We show an example of the power spectrum of a damped harmonic oscillation and its' behaviour on log–log scale in Fig. 33.6. This is a naïve, but necessary demonstration that the harmonic oscillations in the signal should be taken into consideration in order to avoid the meaningless surge of the slope while looking for the power-law trend.

On the other side, He [31] argues that rhythmic, recurring patterns of brain activity in a certain range of frequencies and arrhythmic activity with no prevailing frequency are distinct and the latter would indicate scale-free dynamics. Despite

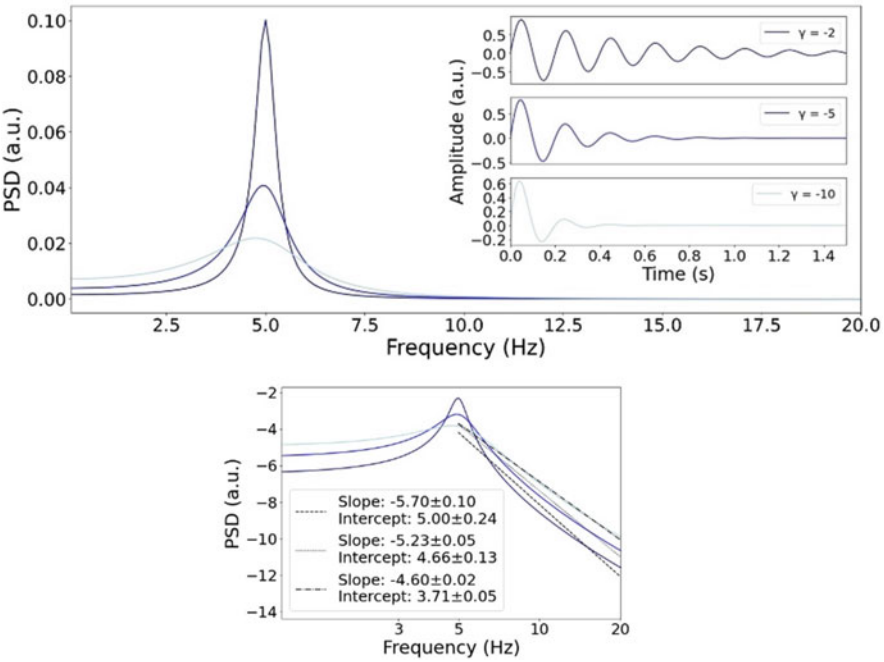

Fig. 33.6 Damped oscillator and artificial surge of a slope that can mimic a power law behaviour. In the top panel, we show the time series (top-right) and the spectrum (top-left) of a simulated periodic damped signal $y(t) = \sin(\omega t)\, e^{-\gamma t}$, with $\omega = 5$ Hz for three values (arbitrarily selected) of the damping parameter $\gamma = -2, -5$ and -10 Hz. In the right panel, the PSD of the signals are given in a log–log scale. The three black straight lines here indicate the best linear fit (evaluated with the least square method) of the higher frequency spectrum portion

that, the author is in line with the idea that periodic oscillations, like those prevailing in the 8–12 Hz frequencies in resting state, generate a trend in the power spectrum that can affect the estimation of the slope β of power law exponent. Therefore, before seeking the fractal nature of an EEG signal, it is necessary to incorporate an approach to identify its class and capture periodic oscillations from those that are irregular and non-periodic. Without this step, the methods deployed to estimate fractal dimension can lead to worthless estimates [21].

33.3.1 Evidence of Existing Scale-Free, Fractal Properties within EEG Signals

Nearly all complex systems express scale-free activity [12, 16, 58]. Despite the claims that $1/f^{\beta}$ can be accounted as a sum of damped harmonic narrow band oscillations [22], the precise patterns in their structure would require the

presence of different underlying mechanisms that generates this specific time-independent activity [32]. Yamamoto and Hughson [56, 57] suggested a method, called coarse-graining spectral analysis (CGSA), to separate physiological signals harmonic oscillations from arrhythmic activity and possibly fractal. Other authors [20] suggested similar techniques for parametrizing periodic and non-periodic oscillatory activity. Alternatively to identify fluctuations in certain time scales one can apply high/low-pass filters [48]. Independently from the method of detrending the harmonic oscillations from time series, the main goal is to obtain the underlying scale-free activity.

Therefore, $1/f^{\beta}$ property rather suggests a scale-free, fractal pattern within the brain. [32] have shown that after removing oscillation from the power spectrum, the power-law exponent β is different in cortical areas to another and changes during different task performances [32]. Furthermore, the authors have proved that 1/f scaling is not an artefact due to the instrumental noise [2], by recording an ECoG in a room without a patient. The results did not demonstrate the power-law distribution of these dummy records.

Interestingly, in the work of He et al. [32] the trend of PSD on a log-log scale was not completely linear, but instead expressed a couple of 'shoulders'. According to their claim, this could occur due to the incomplete elimination of oscillatory components. However, we believe this could be an effect of existing different fractal regimes in the series that yield multifractality. This idea needs further investigation.

Some observations have also suggested that shared noise between two chaotic oscillators can actually induce, rather than inhibit, synchronization [59]. In addition, it was also demonstrated that uncorrelated noise can enhance coherent dynamic activity [43]. The understanding behind this phenomenon can be helpful in looking for the signature of local neurodynamics and synchronization between different cortical parcels. However, since the study of EEG recordings in the frequency domain has its own limits, one might be tempted to look if other estimation methods exist that can catch the non-linearities and fractal nature of the signals.

33.3.2 Fractal Dimension Estimation Methods

The PSD of a gaussian white signal will show a flat behaviour as represented in Fig. 33.7a; on the other hand, the PSD of a fractal signal at each time scale will follow a power law distribution, with a constant slope on a log–log scale, across all frequency ranges, as shown in Fig. 33.7b where the PSD of a Brownian noise signal is represented. However, the electrophysiological time series are the overlap of several signals with different features at different frequency ranges. Moreover, due to the characteristic frequencies in time domain, it becomes more complicated to point out power law properties as the PSD function on log–log scale expresses several peaks and humps as can be seen in Fig. 33.7c. For this reason, it is necessary to find a stable method to investigate the characteristic patterns in temporal scale beyond the frequency domain representation.

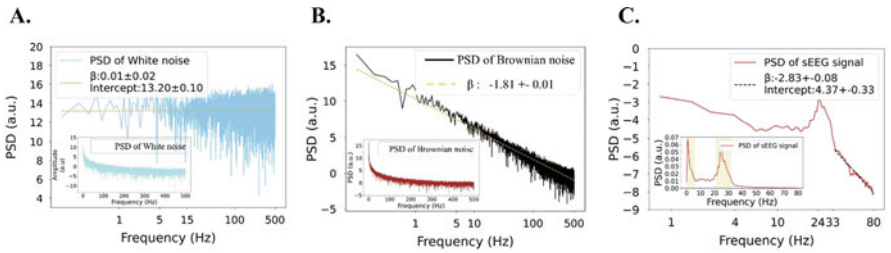

Fig. 33.7 Examples of PSD of white noise, Brownian noise and a sEEG signal. After Armonaite PhD thesis (2022) (**a**) PSD of an example of a typical stationary white noise is given in a double-(semi-) logarithmic scale for the main (inset) figure. The dashed yellow line indicates the best linear fit evaluated with the least square method. (**b**) We show the PSD of a fractal, Brownian noise, that follows power law distribution over all frequency ranges. (**c**) In the inset figure, the PSD of a sEEG signal from primary motor cortex (M1) with clear theta, beta and low gamma peaks is given. In the main figure, the same signal PSD on a log–log scale is plotted with a linear fit over the highest frequency range (33–80 Hz), where the power law behaviour is observed

There are plenty of methods that were used to examine the neurodynamics in healthy and with brain neural activity alterations illnesses subjects. Such as the predictability of ongoing neural activity during cognitive tasks using the Hurst exponent [33], Katz algorithm, correlation dimension and Liapunov exponent for differentiating sleep stages [1, 39], Lempel Ziv and Shannon entropy algorithm for depth of anaesthesia measure [23, 50] and Higuchi fractal dimension as a biomarker of Alzheimer's disease [5, 51]. An attempt to classify cortical areas via the Higuchi fractal dimension is given by [7, 17].

33.3.2.1 Higuchi Fractal Dimension (HFD)

Higuchi fractal dimension (HFD) is considered to be a highly sensitive measure in the detection of information contained in physiological time series [38, 46]. Several authors also claimed that this method has a superiority over other fractal dimension estimation methods [3, 37, 47].

It has been shown that HFD is well assessed in diverse research areas such as analyzing heart rate variability [29], characterizing primary waves in seismograms [27], studying magnetic energy dissipation [55], volcanoes magnetic field [24], geomagnetic field [40] and other electromagnetic physical systems [41].

The reason why HFD would be superior to other fractal dimension estimates is because it is calculated directly in time domain, it can catch signal specificities in short time windows [34] and would be less sensitive to the signals that are not well pre-processed [38].

The method to calculate HFD is as follows.

Consider given N-length time series $\{X(1), X(2), X(3), \ldots, X(N)\}$. For a time interval, we skip a number of samples equal to k. We obtain m number of sets of

new time series $X_k^m : X(m), X(m+k), X(m+2k), \ldots, X(m+[(N-m)/k)])$. Then we calculate the length of the curve X_k^m as follows:

$$L_m(k) = \left[\left(\sum_{i=1}^{\text{int}\left(\frac{N-m}{k}\right)} |X(m+i\,k) - X(m+(i-1)\,k)| \right) \frac{N-1}{\text{int}\left(\frac{N-m}{k}\right) k} \right] \frac{1}{k}$$

The length *L(k)* of the newly generated subseries X_k^m is evaluated by averaging the k sets of $L_m(k)$ values, as:

$$L(k) = \frac{1}{k} \sum_{m=1}^{k} L_m(k)$$

If limit exists for $k \to \infty$, such as $L(k) \propto k^{-\text{HFD}}$ then the curve is fractal with dimension equal to HFD.

Higuchi fractal dimension is a quantitative measure of signal complexity, and it is not independent from power law exponent calculated with the classical spectral analysis. Higuchi [34] in his seminal paper stated, that there is a direct relationship between HFD and the β parameter of the $1/f^{\beta}$ power spectrum that holds $\beta = 5 - 2\text{HFD}$ if $1 \leq \beta \leq 3$ and $1 \leq \text{HFD} \leq 2$.

33.4 Neurodynamics as Local Cortical Signature: A Fractal Estimate

Despite the fact that the power spectrum has been largely applied to estimate the characteristic frequencies of the local neurodynamics, it was demonstrated that the irregular fluctuations of ongoing neuronal activity can be more 'stable' than oscillatory ones: that is, they show less spontaneous intrasubject variability and less intersubject variability [45, 53]. In the papers of Armonaite et al. [7, 8], the authors aimed to demonstrate existing solid neurodynamical patterns across different cortical areas using Higuchi fractal dimension measure during wakefulness and three sleep stages. The purpose of these studies was to deepen knowledge of the brain neurodynamics as cortical area signature. Moving on from the successful attempt to differentiate the neurodynamics of the S1 and M1 hand representations based on noninvasive EEG [17], Armonaite et al. [7, 8] aim to strengthen the results by investigating intracranial sEEG data.

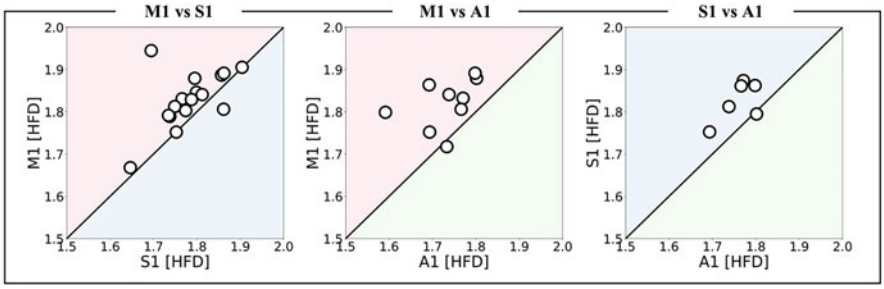

Fig. 33.8 Comparing the HFD of brain areas couples within a subject. After Armonaite et al. [8]. Scatterplots of each subject HFD in couple of areas: S1 versus M1 (16 subjects); A1 versus M1 (9 subjects) and A1 versus S1 (6 subjects). The diagonal is traced indicating where the HFD of the source represented in x is higher/lower than that in y. Colour codes as always indicate sources that are being compared: S1 blue, M1 red and A1 green

33.4.1 Distinct Cortical Areas Exhibit Typical Complexity Relationship During Wakefulness

Investigating the neurodynamics of S1 and M1 cortical parcels during resting wakefulness, sensory stimulation and hand motor function, the estimate of Higuchi fractal dimension shows a cortical area effect with a smaller value for S1 than M1 [17]. Following these results, Armonaite et al. [7] estimated the Higuchi fractal dimension of 21 subject sEEG signals during resting wakefulness in three cortical parcels: A1, S1 and M1. And compared Higuchi fractal dimension between the couples of cortical areas within the subject, initially, and then on average across the population. The results of this analysis are compatible with the previous ones, as it is shown that M1 expresses higher complexity than S1 and both higher than A1 with [$W_{test} = 12$, $p = 0.01$] (see also scatterplot of Fig. 33.8). Then the average value of HFD across the population was evaluated, and authors observed that even though the errors overlap, on average HFD for M1 is higher than S1 and A1 (Fig. 33.9).

33.4.2 Cortical Parcels Hold Typical Fractal Characteristics During Sleep

Since significant differences in neurodynamics across different cortical areas A1, S1 and M1 in resting wakefulness were found, further investigation by Armonaite et al. [8] was done with the aim: (i) to explore if the neurodynamics (expressed by Higuchi fractal dimension) maintain their local specificities along different sleep stages and (ii) to study if the local complexity in the investigated areas reduces in deep sleep with respect to awake and REM stages.

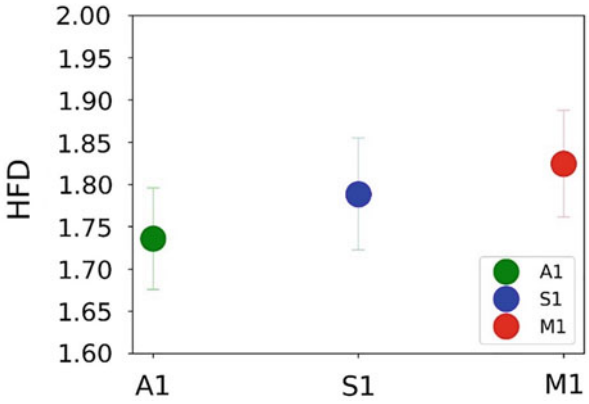

Fig. 33.9 Mean of HFD for the A1, S1 and M1 parcels across population. After Armonaite et al. [8]. The mean and standard deviation HFD evaluated across the population considering only representative channel for the three regions of interest. The HFD value is given for $k_{\max} = 35$

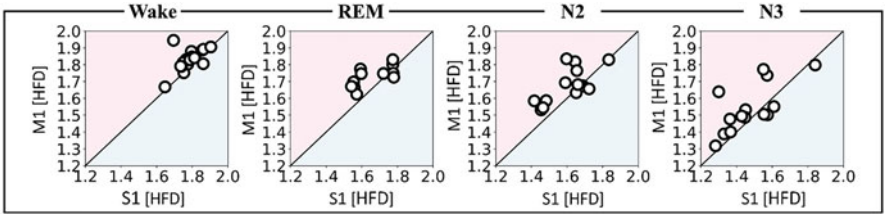

Fig. 33.10 Comparison via HFD of local neurodynamic in pairs of regions. After Armonaite et al. [8]. The scatterplots of HFD between subjects for S1 versus M1 [16 subjects]. In each plot, a point above (below) the diagonal has HFD value of the source represented on the axis lower (higher) than that of the source shown on the axis

The comparisons within the subject showed that across all sleep stages the relationship of Higuchi fractal dimension values in the three investigated cortical parcels shows stable overall behaviour with M1 $\geq$ S1 $\geq$ A1. The comparisons of the HFD values for S1 and M1 within a subject are given in Fig. 33.10.

By taking the mean value of HFD across subjects in each cortical parcel in the three sleep stages as well as wakefulness, an evident decrease of an average HFD value in each cortical parcel (from wakefulness to deeper sleep stages) can be observed (Fig. 33.11).

33.5 Conclusions

1. Evidence – from a selected group of regions – suggests that diverse cortical areas express a typical ongoing local electrical activity as their own signature.
2. Spectral estimation methods may not always reveal the underlying specificities of the signal, failing to adequately account for irregularities and arrhythmias present in the fluctuations. Although Fourier transform estimation allows linkage with current knowledge describing the specific physiological roles of different

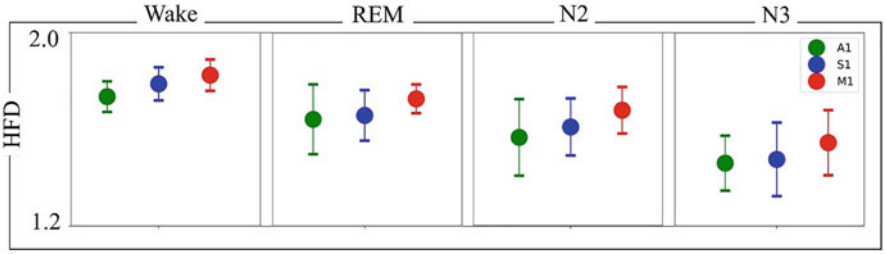

Fig. 33.11 Estimation of complexity, via HFD, of the local neurodynamics across people in wakefulness and sleep stages. After Armonaite et al. [8]. The mean HFD estimated across the population in wakefulness, REM, N2 and N3 sleep stages. For each subject, we considered a representative of all available channels within the A1, S1 and M1 regions. The Higuchi fractal dimension was evaluated at $k_{max} = 35$ in each state

frequency domains, such estimation does not capture the complex nature of fluctuations typical of various cortical parcels.

3. By comparing the neurodynamic signature measured by power spectral density (PSD) and Higuchi fractal dimension (HFD) in wakefulness and during the three sleep phases (REM, N2 and N3), a consistency between the two assessments emerged in three paradigmatic primary cortices (A1, S1 and M1), with M1 exhibiting a higher frequency activity than S1 and A1 not only during the waking state but also in the REM, N2 and N3 sleep phases, albeit in multiple frequency domains that decrease as sleep deepens. The single HFD number shows M1 > S1 > A1 during wakefulness and the three sleep phases, with the value of the three regions decreasing as sleep deepens.
4. The nonlinear measure typical of complex systems, the fractal dimension, emerges as more stable than PSD; a 'synthesis' with a single value of spectral features; useful for classifying sleep stages.

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