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B-parameters for $\Delta S = 2$ supersymmetric operators

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Abstract

We present a calculation of the matrix elements of the most general set of $\Delta S = 2$ dimension-six four-fermion operators. The values of the matrix elements are given in terms of the corresponding B-parameters. Our results can be used in many phenomenological applications, since the operators considered here give important contributions to K^0–$\bar{K}^0$ mixing in several extensions of the Standard Model (supersymmetry, left-right symmetric models, multi-Higgs models etc.). The determination of the matrix elements improves the accuracy of the phenomenological analyses intended to put bounds on basic parameters of the different models, as for example the pattern of the sfermion mass matrices. The calculation has been performed on the lattice, using the tree-level improved Clover action at two different values of the strong coupling constant ($\beta = 6/g_0^2(a) = 6.0$ and 6.2, corresponding to $a^{-1} = 2.1$ and 2.7 GeV respectively), in the quenched approximation. The renormalization constants and mixing coefficients of the lattice operators have been obtained non-perturbatively. © 1999 Published by Elsevier Science B.V. All rights reserved.

1. Introduction

Important information on the physics beyond the Standard Model (SM), such as supersymmetry, left-right symmetric models, multi-Higgs models etc., can be obtained by studying FCNC processes. Among these, $\Delta F = 2$ transitions play a very important role. They have been used in [1,2], for example, to put constraints on the sfermion mass matrix. In this paper we present the results of a lattice calculation of the matrix elements of the most general set of $\Delta S = 2$

dimension-six four-fermion operators, renormalized non-perturbatively in the RI (MOM) scheme [3–7]. Our results can be combined with the recent two-loop calculation of the anomalous dimension matrix in the same renormalization scheme [8] to obtain K^0–$\bar{K}^0$ mixing amplitudes which are consistently computed at the next-to-leading order. A phenomenological application of the results for the matrix elements given below, combined with a complete next-to-leading order (NLO) evolution of the Wilson coefficients, is presented elsewhere [9].

K^0–$\overline{K}^0$ mixing induces the neutral kaon mass difference ΔM_K and is related to the indirect CP violation parameter ϵ_K. In the Standard Model, this transition occurs via the dimension-six four-fermion operator $O^{\Delta S=2}$, with a "left-left"x chiral structure. The B-parameter of the matrix element $\langle \overline{K}^0 | O^{\Delta S=2} | K^0 \rangle$, commonly known as B_K, has been extensively studied on the lattice due to its phenomenological relevance [10], and used in many phenomenological studies [11]. For the other operators, instead, all the phenomenological analyses beyond the SM have used B-parameters equal to one, which in some cases, as will be shown below, is a very crude approximation. The present work is the first fully non-perturbative study of the matrix elements of the complete set of $\Delta S = 2$ four-fermion operators. With respect to other calculations, the systematic errors in our results are reduced in two ways: i) by using the tree-level improved Clover action [12] and operators [13] we obtain matrix elements for which discretization errors are of $\mathcal{O}(\alpha_s a)$; ii) by renormalizing non-perturbatively the lattice operators, we have eliminated the systematic error due to the bad behaviour of lattice perturbation theory. With the non-perturbative renormalization, the residual error is that due to the truncation of the continuum perturbative series in the evaluation of the Wilson coefficients of the effective Hamiltonian. This error is of $\mathcal{O}(\alpha_s^2)$, where α_s is the continuum renormalization constant and could in principle be reduced by the calculation of the N^2LO corrections in the continuum. In the RI scheme, for example, this step has recently been done for the calculation of the renormalized quark mass [14].

For the reader who is not interested in technical details, we now give the results for the B-parameters of the relevant operators. The choice of the basis is arbitrary, and different bases can be found in the literature, see for example [1] and [8]. We have used the SUSY basis for which the numerical values of the Wilson coefficients, computed at the NLO, are given [9], namely [1]

$$O_1 = \bar{s}^\alpha \gamma_\mu (1 - \gamma_5) d^\alpha \, \bar{s}^\beta \gamma_\mu (1 - \gamma_5) d^\beta,$$

$$O_2 = \bar{s}^\alpha (1 - \gamma_5) d^\alpha \, \bar{s}^\beta (1 - \gamma_5) d^\beta,$$

[1] We use here the Euclidean notation.

$$O_3 = \bar{s}^\alpha (1 - \gamma_5) d^\beta \, \bar{s}^\beta (1 - \gamma_5) d^\alpha,$$

$$O_4 = \bar{s}^\alpha (1 - \gamma_5) d^\alpha \, \bar{s}^\beta (1 + \gamma_5) d^\beta,$$

$$O_5 = \bar{s}^\alpha (1 - \gamma_5) d^\beta \, \bar{s}^\beta (1 + \gamma_5) d^\alpha, \tag{1}$$

where α and β are colour indices. The B-parameters for these operators are defined as

$$\langle \overline{K}^0 | \hat{O}_1(\mu) | K^0 \rangle = \frac{8}{3} M_K^2 f_K^2 B_1(\mu),$$

$$\langle \overline{K}^0 | \hat{O}_2(\mu) | K^0 \rangle$$
$$= -\frac{5}{3} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_2(\mu),$$

$$\langle \overline{K}^0 | \hat{O}_3(\mu) | K^0 \rangle$$
$$= \frac{1}{3} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_3(\mu),$$

$$\langle \overline{K}^0 | \hat{O}_4(\mu) | K^0 \rangle$$
$$= 2 \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_4(\mu),$$

$$\langle \overline{K}^0 | \hat{O}_5(\mu) | K^0 \rangle$$
$$= \frac{2}{3} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_5(\mu), \tag{2}$$

where the notation $\hat{O}_i(\mu)$ (or simply $\hat{O}_i$) denotes the operators renormalized at the scale μ. A few words of explanation are necessary at this point. In Eq. (2) operators and quark masses are renormalized at the scale μ in the same scheme (e.g. RI, $\overline{\text{MS}}$, etc.). The numerical results for the B-parameters, $B_i(\mu)$ computed in this paper refer to the RI scheme. Moreover, without loss of generality, we have omitted terms, present in the usual definition of the Bs, which are of higher order in the chiral expansion. Since the definition of the B-parameters is conventional, we prefer to use those in Eq. (2) for which, as explained in Section 3, the scaling properties are the simplest ones.

Our best estimates of the B-parameters, for a renormalization scale of $\mu = 2$ GeV are

$$B_1(\mu) = 0.69 \pm 0.21, \quad B_2(\mu) = 0.66 \pm 0.04,$$
$$B_3(\mu) = 1.05 \pm 0.12, \quad B_4(\mu) = 1.03 \pm 0.06,$$
$$B_5(\mu) = 0.73 \pm 0.10. \tag{3}$$

The remainder of the paper is organized as fol-lows: in Section 2, we address the problem of opera-tor mixing and renormalization and give a brief account of the non-perturbative method (NPM) used in the computation of the operator renormalization constants; in Section 3, we discuss the definition of the B-parameters and describe their extraction from the lattice correlation functions; in Section 4, we present our results for the full operator basis, at $\beta = 6.0$ and 6.2 and for different renormalization scales; a discussion of the errors assigned to the final results in Eq. (3) can also be found in this section; finally, in Section 5, we present our conclusions.

2. Non-Perturbative Renormalization

In this section, we briefly recall the reasons for which the non-perturbative renormalization of the lattice operators is important and describe the proce-dure which has been used to obtain, for the cases of interest, finite matrix elements from the bare lattice operators.

The Wilson lattice regularization breaks chiral symmetry. This implies that each operator in the $\Delta S = 2$ Hamiltonian mixes with operators belonging to different chiral representations [15,16]. Because of the mixing induced by the lattice, the correct chiral behaviour of the operators is achieved with Wilson fermions only in the continuum limit. This repre-sented a long-standing problem in the evaluation of B_K [17,18] [2], only recently solved with the introduc-tion of Non-Perturbative Renormalization methods. In these approaches the renormalization constants (mixing matrix) are computed non-perturbatively on the lattice either by projecting on external quark and gluon states (NPM) as proposed in [3] or, in the spirit of [16], by using chiral Ward Identities [20,21]. Recent studies of the B-parameters, with both non-perturbative renormalization methods, [4–6,21], show that discretization effects are less important than those due to the perturbative evaluation of the mix-ing coefficients.

Given the success of the non-perturbative meth-ods in the computation of B_K, the NPM has been applied to the evaluation of the two $\Delta I = 3/2$ B-parameters of the electro-penguin operators, $B_7^{3/2}$ and $B_8^{3/2}$ (these B-parameters coincide with those of the operators O_4 nd O_5 respectively). Also in this case, as shown in [6], it has been found that the non-perturbative renormalization of the lattice opera-tors gives B-parameters that significantly differ from those renormalized perturbatively [6,22].

An extensive study of the renormalization proper-ties of the four-fermion operators can be found in [7]. There we detail the issues of relevance to the non-perturbative renormalization of all the $\Delta S = 2$ opera-tors. We have used these results in the present study.

The NPM for the evaluation of the renormaliza-tion constants of lattice operators consists in impos-ing suitable renormalization conditions on lattice am-putated quark correlation functions [3]. In our case, we compute four-fermion Green functions in the Landau gauge. All external quark lines are at equal momentum p. After amputating and projecting these correlation functions (see [4] and [7] for details), the renormalization conditions are imposed in the deep Euclidean region at the scale $p^2 = \mu^2$. This renor-malization scheme has been recently called the Reg-ularization Independent (RI) scheme [23] (MOM in the early literature) in order to emphasize that the renormalization conditions are independent of the regularization scheme, although they depend on the external states used in the renormalization procedure (and on the gauge). Thus, at fixed cutoff (i.e. fixed β), we compute non-perturbatively the renormaliza-tion constants and the renormalized operator $\hat{O}^{RI}(\mu)$ in the RI scheme. In order to obtain the physical amplitudes, which are renormalization group invari-ant and scheme independent, the renormalized matrix elements must subsequently be combined with the corresponding Wilson coefficients of the effective Hamiltonian. For the operators of interest, the latter are known at the NLO in continuum perturbation theory [8].

In [7], we have determined non-perturbatively the operator mixing for the complete basis of four-ferm-ion operators, with the aid of the discrete symmetries (parity, charge conjugation and switching of flavours), in the spirit of [17]. The renormalization of the parity-even operators, relevant to this work, is

more conveniently expressed in terms of the following basis of five operators:

$$Q_1 = V \times V + A \times A, \quad Q_2 = V \times V - A \times A,$$
$$Q_3 = S \times S - P \times P, \quad Q_4 = S \times S + P \times P,$$
$$Q_5 = T \times T. \tag{4}$$

The operators $Q_1, \ldots, Q_5$ form a complete basis on the lattice. In these expressions, $\Gamma \times \Gamma$ (with $\Gamma = V, A, S, P, T$ a generic Dirac matrix) stands for $\frac{1}{2}(\overline{\psi}_1 \Gamma \psi_2 \overline{\psi}_3 \Gamma \psi_4 + \overline{\psi}_1 \Gamma \psi_4 \overline{\psi}_3 \Gamma \psi_2)$, where ψ_i, $i = 1, \ldots, 4$ are fermion fields with flavours chosen so as to reproduce the desired operators (see [7] for details).

The parity-even parts of the five SUSY operators defined in Eq. (1), which are the relevant ones for K^0–$\overline{K}^0$ mixing, are related to the operators of Eq. (4) in the following way:

$$O_1 = Q_1, \quad O_2 = Q_4, \quad O_3 = -\tfrac{1}{2}(Q_4 - Q_5),$$
$$O_4 = Q_3, \quad O_5 = -\tfrac{1}{2}Q_2. \tag{5}$$

On the lattice, Q_1 mixes under renormalization with the other four operators as follows

$$\hat{Q}_1 = Z_{11}\left[Q_1 + \sum_{i=2}^{5} Z_{1i} Q_i\right], \tag{6}$$

where Z_{11} is a multiplicative logarithmically divergent renormalization constant; it depends on the coupling and $a\mu$. The mixing coefficients Z_{1i} (with $i = 2, \ldots, 5$) are finite; they only depend on the lattice coupling $g_0^2(a)$.

The other renormalized operators are defined as follows:

$$\hat{Q}_2 = Z_{22} Q_2^s + Z_{23} Q_3^s, \quad \hat{Q}_3 = Z_{32} Q_2^s + Z_{33} Q_3^s,$$
$$\hat{Q}_4 = Z_{44} Q_4^s + Z_{45} Q_5^s, \quad \hat{Q}_5 = Z_{54} Q_4^s + Z_{55} Q_5^s, \tag{7}$$

where the Z_{ij}s are logarithmically divergent renormalization constants which depend on the coupling and $a\mu$. The above mixing matrices are not peculiar to the lattice regularization, but also occur in the continuum. The breaking of chiral symmetry by the Wilson action requires the additional subtractions:

$$Q_i^s = Q_i + \sum_{j=1,4,5} Z_{ij}^s Q_j, \quad i = 2,3,$$
$$Q_i^s = Q_i + \sum_{j=1,2,3} Z_{ij}^s Q_j, \quad i = 4,5,$$

where the Z_{ij}^ss are finite coefficients which only depend on $g_0^2(a)$.

The results for all the renormalization constants Z_{ij} and Z_{ij}^s (computed with the NPM, for several renormalization scales μ, at $\beta = 6.0$ and 6.2) can be found in [7].

3. B-parameters

In this section we discuss the definition of the B-parameters and their dependence on the renormalization scale. We also sketch the extraction of these quantities from lattice correlation functions.

The B-parameters are usually defined as

$$B_i(\mu) = \frac{\langle \overline{K}^0 | \hat{O}_i(\mu) | K^0 \rangle}{\langle \overline{K}^0 | \hat{O}_i | K^0 \rangle_{\text{VSA}}}, \tag{8}$$

where the operator matrix elements in the Vacuum Saturation Approximation (VSA) are given by

$$\langle \overline{K}^0 | \hat{O}_1 | K^0 \rangle_{\text{VSA}} = 2\left(1 + \frac{1}{N_c}\right) |\langle \overline{K}^0 | \hat{A}_\mu | 0 \rangle|^2,$$

$$\langle \overline{K}^0 | \hat{O}_2 | K^0 \rangle_{\text{VSA}} = -2\left(1 - \frac{1}{2N_c}\right) |\langle \overline{K}^0 | \hat{P} | 0 \rangle|^2,$$

$$\langle \overline{K}^0 | \hat{O}_3 | K^0 \rangle_{\text{VSA}} = \left(1 - \frac{2}{N_c}\right) |\langle \overline{K}^0 | \hat{P} | 0 \rangle|^2,$$

$$\langle \overline{K}^0 | \hat{O}_4 | K^0 \rangle_{\text{VSA}} = 2|\langle \overline{K}^0 | \hat{P} | 0 \rangle|^2$$
$$+ \frac{1}{N_c} |\langle \overline{K}^0 | \hat{A}_\mu | 0 \rangle|^2,$$

$$\langle \overline{K}^0 | \hat{O}_5 | K^0 \rangle_{\text{VSA}} = \frac{2}{N_c} |\langle \overline{K}^0 | \hat{P} | 0 \rangle|^2 + |\langle \overline{K}^0 | \hat{A}_\mu | 0 \rangle|^2. \tag{9}$$

$\hat{A}_\mu$ and $\hat{P}$ are the renormalized axial current and pseudoscalar densities, $\hat{A}_\mu = Z_A A_\mu$ and $\hat{P} = Z_P P$, with Z_A the (finite) renormalization constant of the lattice axial current, $A_\mu = \bar{s}\gamma_\mu\gamma_5 d$, and Z_P the renormalization constant of the lattice pseudoscalar density, $P = \bar{s}\gamma_5 d$. For simplicity, $\hat{P}$ is renormalized at the same scale μ, and in the same renormalization

scheme as the four-fermion operators (the RI scheme in our case). Using the relations

$$|\langle \overline{K}^0 | \hat{A}_\mu |0\rangle|^2 = M_K^2 f_K^2 ,$$

$$|\langle \overline{K}^0 | \hat{P} |0\rangle|^2 = \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 , \quad (10)$$

where the second equality is a consequence of the Ward identity for the axial current (with $m_s(\mu)$ and $m_d(\mu)$ renormalized in the same scheme and at the same scale as $\hat{P}$), we find, with $N_c = 3$

$$\langle \overline{K}^0 | \hat{O}_1(\mu) | K^0 \rangle_{\mathrm{VSA}} = \frac{8}{3} M_K^2 f_K^2 ,$$

$$\langle \overline{K}^0 | \hat{O}_2(\mu) | K^0 \rangle_{\mathrm{VSA}}$$

$$= -\frac{5}{3} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 ,$$

$$\langle \overline{K}^0 | \hat{O}_3(\mu) | K^0 \rangle_{\mathrm{VSA}}$$

$$= \frac{1}{3} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 ,$$

$$\langle \overline{K}^0 | \hat{O}_4(\mu) | K^0 \rangle_{\mathrm{VSA}}$$

$$= 2 \left[\left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 + \frac{1}{6} \right] M_K^2 f_K^2 ,$$

$$\langle \overline{K}^0 | \hat{O}_5(\mu) | K^0 \rangle_{\mathrm{VSA}}$$

$$= \frac{2}{3} \left[\left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 + \frac{3}{2} \right] M_K^2 f_K^2 . \quad (11)$$

The VSA values of the matrix elements of $\hat{O}_4$ and $\hat{O}_5$ in Eq. (11) differ from the factors appearing in the definition of the B-parameters in Eq. (2) by the terms proportional to $1/6$ and $3/2$ respectively. These terms, which originate from the squared matrix elements of the axial current in Eq. (9), are of higher order in the chiral expansion and have been dropped in our definition of the B-parameters. This implies that, out of the chiral limit, the values of B_4 and B_5 with our definition differ from those obtained by using Eq. (11). To illustrate this point, let us imagine that as $m_s \to \infty$, for some value of the renormalization scale $\overline{\mu}$, the values of the matrix elements of $\hat{O}_4$ and $\hat{O}_5$ were exactly those of the

VSA. Under these hypotheses, using Eq. (2), we would get $B_4(\overline{\mu}) = 7/6$ and $B_5(\overline{\mu}) = 5/2$ instead of one.

We now explain why we prefer the definition of the B-parameters given in Eq. (2) rather than the standard definition obtained from Eq. (11). Neglecting discretization errors, the B-parameters of the operators $\hat{O}_2 - \hat{O}_5$ defined in Eq. (2) obey the renormalization group equation

$$\mu \frac{dB_i(\mu)}{d\mu} = \left(\gamma_{O_i} - 2\gamma_P \right) B_i(\mu) , \quad (12)$$

where $\mu d/d\mu = \mu \partial/\partial\mu + \beta(\alpha_s)\partial/\partial\alpha_s$, and γ_{O_i} ($\gamma_{O_i O_j}$) and γ_P are the anomalous dimension (matrix) of the operator $\hat{O}_i(\mu)$ and of the scalar density respectively [3]. The physical amplitude is given by

$$\langle \overline{K}^0 | \mathscr{H}_{eff} | K^0 \rangle = C_i(M_W/\mu) \langle \overline{K}^0 | \hat{O}_i(\mu) | K^0 \rangle$$

$$= C_i(M_W/\mu) \times B_i(\mu)$$

$$\times \frac{1}{(m_s(\mu) + m_d(\mu))^2} M_K^4 f_K^2$$

$$\sim \left(\frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right)^{-\gamma_{O_i}/2\beta_0}$$

$$\times \left(\alpha_s(\mu) \right)^{(\gamma_{O_i} - 2\gamma_P)/2\beta_0}$$

$$\times \left(\alpha_s(\mu) \right)^{\gamma_P/\beta_0} , \quad (13)$$

where, in the last expression, we have only shown the leading behaviour of the different factors which depend on μ, namely the Wilson coefficient, the B-parameter and the quark masses. Eq. (13) shows explicitly the cancellation of the μ-dependent terms in the amplitude: the quark masses scale with an anomalous dimension which is opposite in sign to that of the pseudoscalar density (since $m(\mu)\hat{P}(\mu)$ is renormalization group invariant) so that $B_i(\mu)/m^2(\mu)$ scales as the corresponding operator $\hat{O}_i(\mu)$; the μ-dependence of the latter is then cancelled by that of the corresponding Wilson coefficient. This remains true at all order in α_s. Out of the chiral limit, with the standard definition of the B-parame-

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[3] For simplicity we ignore the mixing of the operators $\hat{O}_2 - \hat{O}_3$ and $\hat{O}_4 - \hat{O}_5$.

ters obtained by using the VSA matrix elements of Eq. (11), the scaling properties of $B_4(\mu)$ and $B_5(\mu)$ would have been much more complicated. The reason is that, in these cases, the two contributions on the right hand side have a piece which scales as the squared pseudoscalar density and another (proportional to the physical quantity $|\langle \bar{K}^0 | \hat{A}_\mu | 0 \rangle|^2$) which is renormalization group invariant. The μ-independence of the final result would then have been recovered then in a very intricate way. Since the definition of the B-parameters is conventional, we prefer to use that of Eq. (2), for which the scaling properties of all the B-parameters are the simplest ones. Moreover, with this choice, they are the same as those derived in the chiral limit.

In order to extract the B-parameters, we need to compute the following two- and three-point correlation functions:

$$G_P(t_x, \boldsymbol{p}) = \sum_{\boldsymbol{x}} \langle P(x) P^\dagger(0) \rangle e^{-\boldsymbol{p} \cdot \boldsymbol{x}},$$

$$G_A(t_x, \boldsymbol{p}) = \sum_{\boldsymbol{x}} \langle A_0(x) P^\dagger(0) \rangle e^{-\boldsymbol{p} \cdot \boldsymbol{x}},$$

$$G_{\hat{O}}(t_x, t_y; \boldsymbol{p}, \boldsymbol{q})$$
$$= \sum_{\boldsymbol{x}, \boldsymbol{y}} \langle P^\dagger(y) \hat{O}(0) P^\dagger(x) \rangle e^{-\boldsymbol{p} \cdot \boldsymbol{y}} e^{\boldsymbol{q} \cdot \boldsymbol{x}}, \qquad (14)$$

where $x \equiv (\boldsymbol{x}, t_x)$, $y \equiv (\boldsymbol{y}, t_y)$ and $\hat{O}$ stands for any renormalized four-fermion operator of interest. All correlation functions have been evaluated with degenerate quark masses. By forming suitable ratios of the above correlations, and looking at their asymptotic behaviour at large time separations, we can isolate the desired matrix elements

$$R_1 = \frac{G_{\hat{O}_1}}{Z_A^2 G_P G_P} \to \frac{\langle \bar{K}^0(\boldsymbol{q}) | \hat{O}_1 | K^0(\boldsymbol{p}) \rangle}{Z_A^2 |\langle 0 | P | K^0 \rangle|^2},$$

$$R_2 = -\frac{1}{2 \left(1 - \dfrac{1}{2N_c}\right) Z_P^2} \frac{G_{\hat{O}_2}}{G_P G_P}$$

$$\to -\frac{1}{2 \left(1 - \dfrac{1}{2N_c}\right)} \frac{\langle \bar{K}^0(\boldsymbol{q}) | \hat{O}_2 | K^0(\boldsymbol{p}) \rangle}{Z_P^2 |\langle 0 | P | K^0 \rangle|^2},$$

$$R_3 = \frac{1}{\left(1 - \dfrac{2}{N_c}\right) Z_P^2} \frac{G_{\hat{O}_3}}{G_P G_P}$$

$$\to \frac{1}{\left(1 - \dfrac{2}{N_c}\right)} \frac{\langle \bar{K}^0(\boldsymbol{q}) | \hat{O}_3 | K^0(\boldsymbol{p}) \rangle}{Z_P^2 |\langle 0 | P | K^0 \rangle|^2},$$

$$R_4 = \frac{1}{2 Z_P^2} \frac{G_{\hat{O}_4}}{G_P G_P} \to \frac{1}{2} \frac{\langle \bar{K}^0(\boldsymbol{q}) | \hat{O}_4 | K^0(\boldsymbol{p}) \rangle}{Z_P^2 |\langle 0 | P | K^0 \rangle|^2},$$

$$R_5 = \frac{N_c}{2 Z_P^2} \frac{G_{\hat{O}_5}}{G_P G_P} \to \frac{N_c}{2} \frac{\langle \bar{K}^0(\boldsymbol{q}) | \hat{O}_5 | K^0(\boldsymbol{p}) \rangle}{Z_P^2 |\langle 0 | P | K^0 \rangle|^2}.$$
$$(15)$$

We stress that the B-parameters extracted from R_1, R_4 and R_5 are identical to the B-parameters for the operators $O^{\Delta S = 2}, O_8^{3/2}$ and $O_7^{3/2}$ respectively. In [6], the results referred to the operators $O^{\Delta S = 2}, O_8^{3/2}$ and $O_7^{3/2}$ at $\beta = 6.0$ only. In this paper, we present the results for all the B-parameters and for $\beta = 6.0$ and 6.2.

4. Numerical results

Our simulations have been performed at $\beta = 6.0$ and 6.2 with the tree-level Clover action, for several values of the quark masses (corresponding to the

Table 1
Summary of the parameters of the runs at $\beta = 6.0$ (run A) and $\beta = 6.2$ (run B). The calibration of the lattice spacing a^{-1} has been done using the lattice-plane method of [25]

Parameter	Run A	Run B
β	6.0	6.2
No. Confs.	460	200
Volume	$18^3 \times 64$	$24^3 \times 64$
k	0.1425	0.14144
	0.1432	0.14184
	0.1440	0.14224
	–	0.14264
Time intervals	10–22	14–26
	42–54	38–50
$a^{-1}(K^*)$ (GeV)	2.12(4)	2.7(1)

Table 2
Values of the B-parameters in the chiral limit, at different scales $\mu^2 a^2$ for $\beta = 6.0$

$\mu^2 a^2$	B_1	B_2	B_3	B_4	B_5
0.31	0.72 ± 0.14	0.70 ± 0.08	1.62 ± 0.28	1.21 ± 0.08	1.70 ± 0.24
0.62	0.70 ± 0.15	0.64 ± 0.04	1.21 ± 0.12	1.08 ± 0.05	0.81 ± 0.10
0.96	0.70 ± 0.15	0.61 ± 0.03	1.10 ± 0.08	1.04 ± 0.04	0.68 ± 0.07
1.27	0.70 ± 0.15	0.60 ± 0.03	1.03 ± 0.07	1.01 ± 0.04	0.69 ± 0.06
1.38	0.70 ± 0.15	0.59 ± 0.03	1.03 ± 0.07	1.01 ± 0.04	0.70 ± 0.05
1.85	0.71 ± 0.15	0.59 ± 0.03	0.98 ± 0.06	0.99 ± 0.04	0.68 ± 0.05
2.46	0.72 ± 0.15	0.57 ± 0.03	0.96 ± 0.05	1.00 ± 0.04	0.69 ± 0.04
4.00	0.74 ± 0.15	0.55 ± 0.02	0.87 ± 0.04	0.99 ± 0.03	0.73 ± 0.04

values of the hopping parameter k given in table Table 1), in the quenched approximation. The physical volume is approximatively the same on the two lattices. A summary of the main parameters is given in the same table. ''Time Intervals'' denote the range in time (in lattice units) on which the two-point correlation functions have been fitted to extract the meson masses and the matrix elements of A_μ and P. The ratios R_i, related to the matrix elements of the four-fermion operators, have been extracted on the same time intervals. Statistical errors have been estimated with the jacknife method, by decimating 10 configurations at a time.

As discussed in Section 2, the renormalization constants have been obtained from the quark correlation functions, in the Landau gauge. The results for the Zs have been obtained on a $16^3 \times 32$ ($16^3 \times 32$) lattice at $\beta = 6.0$ (6.2), using a statistical sample of 100 (180) configurations. In constructing the renormalized operators we have used the central values of the renormalization constants neglecting their statistical errors. For this reason the errors on the B-parameters only include those of operator matrix

elements. In the ratios (Eq. (15)), we also need the axial-current renormalization constant Z_A and the μ-dependent renormalization constant Z_P of the pseudoscalar density. Although Z_A should not depend on $a\mu$, slight variations of its NPM estimate, arising from systematic effects, partially cancel analogous variations of R_1, giving more stable results in the extraction of the matrix elements. The NPM estimates for Z_P and Z_A used in the present work are those of [24].

In order to extract the B-parameters from the ratios of Eq. (15), we follow the procedure of [5], by fitting the R_i linearly in X and Y (i.e. linearly in m_K^2 and $(p \cdot q)$) with the function

$$R_i = \alpha_i + \beta_i X + \gamma_i Y, \tag{16}$$

where

$$X = \frac{8}{3} \frac{G_A G_A^\dagger}{G_P G_P} \rightarrow \frac{8}{3} \frac{f_K^2 m_K^2}{Z_A^2 |\langle 0|P|K^0 \rangle|^2},$$

$$Y = \frac{(p \cdot q)}{m_K^2} X, \tag{17}$$

Table 3
Values of the B-parameters in the chiral limit, at different scales $\mu^2 a^2$ for $\beta = 6.2$

$\mu^2 a^2$	B_1	B_2	B_3	B_4	B_5
0.31	0.67 ± 0.21	0.72 ± 0.10	0.90 ± 0.40	0.99 ± 0.12	0.21 ± 0.24
0.62	0.68 ± 0.21	0.63 ± 0.06	0.94 ± 0.16	0.98 ± 0.08	0.46 ± 0.13
0.96	0.68 ± 0.21	0.60 ± 0.04	0.91 ± 0.11	0.95 ± 0.07	0.54 ± 0.10
1.27	0.68 ± 0.21	0.58 ± 0.04	0.89 ± 0.08	0.93 ± 0.07	0.56 ± 0.09
1.38	0.68 ± 0.21	0.58 ± 0.04	0.88 ± 0.08	0.92 ± 0.07	0.55 ± 0.09
1.85	0.67 ± 0.21	0.57 ± 0.03	0.84 ± 0.07	0.91 ± 0.06	0.56 ± 0.08
2.46	0.68 ± 0.21	0.56 ± 0.03	0.82 ± 0.06	0.91 ± 0.06	0.58 ± 0.07
4.00	0.69 ± 0.21	0.54 ± 0.03	0.78 ± 0.05	0.91 ± 0.06	0.60 ± 0.07

Table 4
Values of the *B*-parameters at the physical kaon mass, at different scales $\mu^2 a^2$ for $\beta = 6.0$

$\mu^2 a^2$	B_1	B_2	B_3	B_4	B_5
0.31	0.72 ± 0.14	0.74 ± 0.06	1.57 ± 0.22	1.19 ± 0.06	1.70 ± 0.19
0.62	0.70 ± 0.15	0.69 ± 0.03	1.22 ± 0.10	1.08 ± 0.04	0.92 ± 0.09
0.96	0.70 ± 0.15	0.66 ± 0.03	1.12 ± 0.07	1.05 ± 0.03	0.79 ± 0.06
1.27	0.70 ± 0.15	0.65 ± 0.02	1.06 ± 0.06	1.03 ± 0.03	0.79 ± 0.05
1.38	0.70 ± 0.15	0.64 ± 0.02	1.06 ± 0.05	1.02 ± 0.03	0.79 ± 0.05
1.85	0.71 ± 0.15	0.63 ± 0.02	1.02 ± 0.05	1.01 ± 0.03	0.77 ± 0.04
2.46	0.72 ± 0.15	0.61 ± 0.02	0.99 ± 0.04	1.02 ± 0.03	0.77 ± 0.04
4.00	0.74 ± 0.15	0.59 ± 0.02	0.90 ± 0.03	1.01 ± 0.03	0.81 ± 0.03

with

$$p \cdot q = E(p) E(q) - \mathbf{p} \cdot \mathbf{q},$$

$$\sinh^2\left(\frac{E(p)}{2}\right) = \sinh^2\left(\frac{m}{2}\right) + \sum_{i=1,3} \sin^2\left(\frac{p_i}{2}\right). \tag{18}$$

Assuming α_1 and β_1 to be zero, B_1 is given by:

$$B_1 = \gamma_1 . \tag{19}$$

Since we are working in the linear approximation in X and Y, there is no difference between the value obtained in the chiral limit and at the physical kaon mass. In the chiral limit, the *B*-parameters for the other four operators are given by

$$B_i = \alpha_i, \qquad (i = 2,3,4,5) . \tag{20}$$

We stress again that $B_4 = B_8^{3/2}$ and $B_5 = B_7^{3/2}$. At the physical kaon mass we have instead

$$B_i = \alpha_i + (\beta_i + \gamma_i) X_s, \qquad (i = 2,3,4,5), \tag{21}$$

where X_s is obtained by extrapolating linearly X as a function of the squared pseudoscalar meson mass to the physical value $m_K^{\exp}$.

In Tables 2 and 3 we give the values of the *B*-parameters in the chiral limit, extracted using Eq. (19) and Eq. (20), at $\beta = 6.0$ and 6.2 respectively; in tables Tables 4 and 5 the *B*-parameters are evaluated at the physical kaon mass using Eq. (21) for $i = 2,3,4,5$. At $\beta = 6.0$, the results for B_1, B_4 and B_5 extrapolated to the chiral limit are slightly different from those of [6]. There are several reasons for the differences: i) in order to fix the scale and the strange quark mass we have used the lattice-plane method of [25]; ii) in the present analysis, we use the ''lattice dispersion relation'' of Eq. (18), instead

than the continuum one $E^2 = m^2 + \sum_{i=1,3} p_i^2$ which was adopted in our previous study [4]; iii) in order to reduce the systematic effects due to higher order terms in the chiral expansion, i.e. to higher powers of $p \cdot q$, we have not used the results corresponding to $\mathbf{p} = 2\pi/L(1,0,0)$ and $\mathbf{q} = 2\pi/L(-1,0,0)$. This choice stabilizes the results for B_1 between $\beta = 6.0$ and $\beta = 6.2$ whilst the results for the other *B*-parameters remain essentially unchanged.

We now describe the criteria followed in order to obtain our best estimates of the *B*-parameters. Although we have data at two different values of the lattice spacing, the statistical errors, and the uncertainties in the extraction of the matrix elements, are too large to enable any extrapolation to the continuum limit $a \to 0$: within the precision of our results we cannot detect the dependence of *B*-parameters on a. For this reason, we estimate the central values by averaging the *B*-parameters obtained with the physical mass m_K^{exp} at the two values of β. Since the results at $\beta = 6.0$ have smaller statistical errors but suffer from larger discretization effects, we do not weight the averages with the quoted statistical errors but take simply the sum of the two values divided by two. As far as the errors are concerned we take the largest of the two statistical errors. This is a rather conservative way of estimating the errors. In order to compare the results of Run A and Run B, we must choose the same physical renormalization scale μ.

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[4] Although the continuum and lattice dispersion relations are equivalent at the order in a at which we are working, the latter gives a better description of the data for large momenta, see for example [26].

Table 5
Values of the B-parameters at the physical kaon mass, at different scales $\mu^2 a^2$ for $\beta = 6.2$

$\mu^2 a^2$	B_1	B_2	B_3	B_4	B_5
0.31	0.67 ± 0.21	0.75 ± 0.07	1.05 ± 0.29	1.05 ± 0.09	0.57 ± 0.18
0.62	0.68 ± 0.21	0.66 ± 0.04	0.98 ± 0.12	1.01 ± 0.06	0.67 ± 0.10
0.96	0.68 ± 0.21	0.63 ± 0.03	0.95 ± 0.08	0.99 ± 0.06	0.70 ± 0.08
1.27	0.68 ± 0.21	0.61 ± 0.03	0.92 ± 0.07	0.97 ± 0.05	0.71 ± 0.07
1.38	0.68 ± 0.21	0.61 ± 0.03	0.91 ± 0.06	0.97 ± 0.05	0.70 ± 0.07
1.85	0.67 ± 0.21	0.60 ± 0.03	0.88 ± 0.06	0.97 ± 0.05	0.70 ± 0.06
2.46	0.68 ± 0.21	0.59 ± 0.03	0.86 ± 0.05	0.97 ± 0.05	0.71 ± 0.06
4.00	0.69 ± 0.21	0.57 ± 0.02	0.82 ± 0.04	0.97 ± 0.05	0.73 ± 0.06

Using the estimates of the lattice spacing given in table Table 1, we have taken $\mu^2 a^2 = 0.96$ and $\mu^2 a^2 = 0.62$, corresponding to $\mu = 2.08$ GeV and $\mu = 2.12$ GeV, at $\beta = 6.0$ and 6.2 respectively. We quote the results as obtained at $\mu = 2$ GeV, since the running of the matrix elements between $\mu \sim 2.1$ and 2.0 is totally negligible in comparison with the final errors. In table Table 6, we summarize the values which have been used to give the final estimates. The columns denoted by m_K^{exp} have been used to get the final results in Eq. (3).

In [22] B_2 and B_3 have been obtained at $\beta = 6.0$ with the Wilson action and the operators renormalized perturbatively in the $\overline{\text{MS}}$ scheme; the result is

$$B_2 = 0.59 \pm 0.01, \quad B_3 = 0.79 \pm 0.01 \qquad (22)$$

Although a direct comparison is not possible (our results are in the RI scheme), to the extent that the matching coefficients between the two schemes are a small effect [6], comparison of Eq. (3) and Eq. (22) suggests that perturbative renormalization gives sig-nificantly different results in some cases. This confirms the need for non perturbative renormalization.

5. Conclusions

In this paper we have presented a lattice calculation of the matrix elements of the most general set of $\Delta S = 2$ dimension-six four-fermion operators, renormalized non-perturbatively in the RI (MOM) scheme [3–7]. The calculations have been performed at two different values of the lattice spacing a. Although our precision is not sufficient to make an extrapolation to the continuum limit, the comparison between the results on two different lattices allows a better estimate of the final errors. The main results for the five B-parameters are summarized in Table 6. From this table, we have extracted our best estimates which are given in Eq. (3). We observe that the lattice values of $B_{3,4}$ are close to their VSA whereas this is not true for $B_{1,2,5}$. Our results allow an improvement in the accuracy of phenomenological analyses intended to put bounds on basic parameters of theories beyond the Standard Model.

Table 6
B-parameters at the renormalization scale $\mu = a^{-1} \simeq 2$ GeV, corresponding to $\mu^2 a^2 = 0.96$ and $\mu^2 a^2 = 0.62$ at $\beta = 6.0$ and 6.2 respectively. All results are in the RI (MOM) scheme

$B(\mu \simeq 2$ GeV)	$\beta = 6.0$			$\beta = 6.2$	
	$m_K = 0$ [6]	$m_K = 0$ this work	$m_K = m_K^{exp}$ this work	$m_K = 0$ this work	$m_K = m_K^{exp}$ this work
B_1	0.66(11)	0.70(15)	0.70(15)	0.68(21)	0.68(21)
B_2	--	0.61(3)	0.66(3)	0.63(6)	0.66(4)
B_3	--	1.10(8)	1.12(7)	0.94(16)	0.98(12)
B_4	1.03(3)	1.04(4)	1.05(3)	0.98 (8)	1.01(6)
B_5	0.72(5)	0.68(7)	0.79(6)	0.46(13)	0.67(10)

References

[1] F. Gabbiani, E. Gabrielli, A. Masiero, L. Silvestrini, Nucl. Phys. B 477 (1996) 321.

[2] J. Bagger, K. Matchev, R. Zhang, Phys. Lett. B 412 (1997) 77.

[3] G. Martinelli et al., Nucl. Phys. B 445 (1995) 81.

[4] A. Donini et al., Phys. Lett. B 360 (1996) 83.

[5] M. Crisafulli et al., Phys. Lett. B 369 (1996) 325.

[6] L. Conti et al., Phys. Lett. B 421 (1998) 273.

[7] A. Donini, V. Gimenez, G. Martinelli, M. Talevi, A. Vladikas, prep. ROME1-1181/97; hep-lat/9902030.

[8] M. Ciuchini et al., Nucl. Phys. B 523 (1998) 501.

[9] M. Ciuchini et al., JHEP 10 (1998) 008.

[10] S.R. Sharpe, in: Proc. Lattice 96, 14th Int. Symp. on Lattice Field Theory, St. Louis, USA, 1996, Nucl. Phys. B (Proc. Suppl.) 53 (1997) 181.

[11] M. Lusignoli, L. Maiani, G. Martinelli, L. Reina, Nucl. Phys. B 369 (1992) 139; M. Ciuchini et al., Z. Phys. C 68 (1995) 239; A.J. Buras, M. Jamin, M.E. Lautenbacher, Phys. Lett. B 389 (1996) 749; A. Ali, D. London, Nucl. Phys. A (Proc. Suppl.) 54 (1997) 297; P. Paganini, F. Parodi, P. Roudeau, A. Stocchi, Phys. Scripta 58 (1998) 556.

[12] B. Sheikholeslami, R. Wohlert, Nucl. Phys. B 259 (1985) 572.

[13] G. Heatlie et al., Nucl. Phys. B 352 (1991) 266.

[14] E. Franco, V. Lubicz, Nucl. Phys. B 531 (1998) 641.

[15] G. Martinelli, Phys. Lett. B 141 (1984) 395; C. Bernard, T. Draper, A. Soni, Phys. Rev. D 36 (1987) 3224.

[16] M. Bochicchio et al. Nucl. Phys. B 262 (1985) 331; L. Maiani et al., Phys. Lett. B 176 (1986) 445; Nucl. Phys. 289 (1987) 505.

[17] C. Bernard et al., Nucl. Phys. B (Proc. Suppl.) 4 (1988) 483.

[18] M.B. Gavela et al., Nucl. Phys. B 306 (1988) 677; B (Proc. Suppl.) 17 (1990) 769; C. Bernard, A. Soni, Nucl. Phys. B (Proc. Suppl.) 17 (1990) 495; B (Proc. Suppl.) 42 (1995) 391; R. Gupta et al., Phys. Rev. D 47 (1993) 5113; T. Bhattacharya, R. Gupta, Nucl. Phys. B (Proc. Suppl.) 42 (1995) 935.

[19] S.R. Sharpe, Nucl. Phys. B (Proc. Suppl.) 34 (1994) 403; N. Ishizuka et al., Phys. Rev. Lett. 71 (1993) 24; Nucl. Phys. B (Proc. Suppl.) 34 (1994) 403; G. Kilcup, R. Gupta, S.R. Sharpe, Phys. Rev. D 57 (1998) 1654; S. Aoki et al., Nucl. Phys. B (Proc. Suppl.) 53 (1997) 341.

[20] G. Martinelli et al., Phys. Lett. B 311 (1993) 241; B 317 (1993) 660 (E); M. Paciello et al., Phys. Lett. B 341 (1994) 187.

[21] JLQCD Collaboration, S. Aoki et al., hep-lat/9705035; Nucl. Phys. B (Proc. Suppl.) 53 (1997) 349.

[22] T. Bhattacharya, R. Gupta, S. Sharpe, Phys. Rev. D 55 (1997) 4036.

[23] M. Ciuchini et al., Z. Phys. C 68 (1995) 239.

[24] V. Gimenez, L. Giusti, F. Rapuano, M. Talevi, Nucl. Phys. B 531 (1998) 429; B 540 (1999) 472.

[25] C.R. Allton, V. Gimenez, L. Giusti, F. Rapuano, Nucl. Phys. B 489 (1997) 427.

[26] L. Lellouch et al., Nucl. Phys. B 444 (1995) 401.