Heavy-Light Meson Decay Constants on the Lattice

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We present a high statistics study of the D- and B-meson decay constants. The results were obtained by using the Clover and Wilson lattice actions at $\beta = 6.0$ and 6.2.

1. INTRODUCTION

f_B is a relevant parameter in the determination of the CKM matrix elements and in the study of B-$\overline{B}$ mixing. In this paper, we present the results of a high statistics study of the heavy-light decay constants, at $\beta = 6.0$ and $\beta = 6.2$, with the Wilson and the SW-Clover actions [1] in the quenched approximation. The main parameters and the details of the simulations are given in [2].

We extract the raw lattice value of f_P, f_P^{latt}, using the usual ratio method

$$f_P^{latt} = \left\langle \frac{<A_0 P^\dagger>(t)}{<PP^\dagger>(t)} \coth\left(M_P\left(\frac{T}{2}-t\right)\right) \right\rangle \frac{\sqrt{Z_{PP}}}{M_P},$$

with $P = \overline{Q}(x)\gamma_5 q(x)$ and $A_0 = \bar{Q}(x)\gamma_0\gamma_5 q(x)$, where Q and q denote the heavy and light quark fields respectively and $\langle...\rangle$ is a weighted average over a given time interval t_1-t_2. M_P (the pseudoscalar meson mass) and Z_{PP} are extracted from a fit of $<PP^\dagger>(t)$ as a function of t. The physical value of f_P is then simply given by

$$f_P \equiv \frac{<0|A_0|P(\vec{p}=0)>}{M_P} = f_P^{latt} \, Z_A(a) \, a^{-1}. \quad (1)$$

where Z_A is the renormalization constant of the axial current. Alternatively, we can extract the decay constant of the heavier mesons, by normalizing it to f_π (or to f_K), defined as the pseudoscalar decay constant computed in the chiral limit

$$f_P = R_P \times f_\pi^{exp} = \frac{f_P^{latt}(M_P)}{f_\pi^{latt}} \times f_\pi^{exp}, \quad (2)$$

where $R_P \equiv f_P^{latt}/f_\pi^{latt}$ and $f_\pi^{latt} = f_P^{latt}(M_P = 0)$. We also introduce $R_{P_s} = f_{P_s}^{latt}/f_K^{latt}$ for the meson with the strange quark. We have then $f_{P_s} = R_{P_s} \times f_K^{exp}$.

In order to obtain the physical values of f_D, f_{D_s}, etc., we have to extrapolate f_P both in the heavy and light quark masses. To be specific, we consider R_{D_s}, obtained from a linear fit in the light quark mass and (then) from a fit in the heavy quark mass (at fixed m_s) of the form

$$f_{P_s}\sqrt{M_{P_s}} \approx \Phi_s^{inf} + \frac{\Phi_s'}{M_{P_s}} + \frac{\Phi_s''}{M_{P_s}^2} + \cdots, \quad (3)$$

Φ_s^{inf}, Φ_s', Φ_s'' are functions which are expected to depend logarithmically on m_H but have been taken constant in the fit.

The major sources of uncertainty in the determination of f_P, besides the quenched approximation, come from the calculation of Z_A in eq. (1) and from discretization errors of $O(a)$. The use of chiral Ward identities for a non-perturbative determination of Z_A [3], and the "improved" lattice actions [1,4] can help us to reduce these sources of errors. Another method to get rid of Z_A consists of extracting the decay constants of heavier pseudoscalar mesons by multiplying R_P by the experimental value of the pion decay constant.

Comparing the results from two different actions we studied the reduction of the discretization errors in the improved case and verified the validity of some KLM prescriptions [5,6] that have been proposed to correct $O(a)$ effects in the Wilson case. We corrected f_P^{latt} for any given pair of values of the quark masses $(m_{1,2})$, by multiplying it by the factor

$$\mathcal{F}_{KLM}^W = \sqrt{(1 + am_1)(1 + am_2)} \text{ , in the Wil-}$$

Run		C60	C62	W60	W62a	W62b
β		6.0	6.2	6.0	6.2	6.2
Action		SW	SW	Wil	Wil	Wil
# Confs		170	250	120	250	110
Volume		$18^3 \times 64$	$24^3 \times 64$	$18^3 \times 64$	$24^3 \times 64$	$24^3 \times 64$
$R_{D_s} =$	linear	1.56(3)	1.48(6)	1.11(3)	1.23(4)	1.19(5)
	quadratic	1.57(4)	1.49(7)	1.13(4)	1.25(5)	1.20(5)
f_{D_s}/f_K	linear KLM	1.59(3)	1.50(6)	1.48(6)	1.52(5)	1.47(6)
	quadratic KLM	1.61(4)	1.51(7)	1.51(5)	1.55(7)	1.47(6)
$R_D =$	linear	1.63(4)	1.58(8)	1.14(4)	1.31(6)	1.25(7)
	quadratic	1.69(8)	1.73(16)	1.19(8)	1.43(11)	1.28(9)
f_D/f_π	linear KLM	1.67(4)	1.60(8)	1.52(5)	1.61(7)	1.53(8)
	quadratic KLM	1.72(8)	1.75(16)	1.59(10)	1.77(13)	1.59(11)
f_{D_s}/f_D	linear	1.08(1)	1.07(2)	1.06(1)	1.07(1)	1.09(2)
	quadratic	1.09(3)	1.04(4)	1.06(3)	1.06(3)	1.13(3)
	linear	1.48(7)	1.28(9)	0.79(4)	0.83(4)	0.84(5)
f_{B_s}/f_K	quadratic	1.49(9)	1.27(12)	0.81(5)	0.84(5)	0.84(5)
	linear KLM	1.56(7)	1.33(9)	1.29(4)	1.26(6)	1.16(7)
	quadratic KLM	1.57(12)	1.33(11)	1.33(6)	1.27(8)	1.16(7)
	linear	1.53(9)	1.32(13)	0.81(5)	0.86(6)	0.86(6)
f_B/f_π	quadratic	1.57(16)	1.37(66)	0.87(9)	0.91(11)	0.86(9)
	linear KLM	1.61(10)	1.38(13)	1.32(6)	1.31(8)	1.19(8)
	quadratic KLM	1.65(32)	1.43(50)	1.41(11)	1.39(15)	1.19(11)
f_{B_s}/f_B	linear	1.10(3)	1.14(6)	1.05(2)	1.10(3)	1.12(3)
	quadratic	1.13(7)	1.17(17)	1.03(6)	1.14(6)	1.20(7)

Table 1

Summary of the physical results for $R_{D_s} = f_{D_s}/f_K$, $R_D = f_D/f_\pi$ and for f_{B_s}/f_K, f_B/f_π obtained by extrapolating R_P and R_{P_s}. We also give f_{D_s}/f_D and f_{B_s}/f_B. "linear" and "quadratic" refer to the fit in the light quark masses.

son case and by the factor [7] $\mathcal{F}^C_{KLM} = \frac{\mathcal{F}^W_{KLM}}{F_1 F_2}$, in the Clover case, where

$$F_{1,2} = 1 + \tfrac{1}{4}\left[(1 + am_{1,2}) - (1 + am_{1,2})^{-1}\right].$$

2. PHYSICAL RESULTS

In table 1 are reported the results obtained by fitting R_{P_s} (R_P) to eq. (3) both for a linear and for a quadratic fit in the light quark masses. The scale value has been fixed from the sting tension σ [8]. Although R_{D_s} is a dimensionless quantity, the calibration of the lattice spacing can affect its value because it enters in the determination of the values of the quark masses at which we extrapolate R_{P_s}. However we find the error due to the calibration of the lattice spacing negligible for this ratio. The same is true for the differents methods to fix the strange quark mass value.

Without KLM factors the results in the Wilson

case are incompatible with those obtained with the Clover action, but we note the remarkable agreement between the scaled KLM-Wilson and the Clover data at $\beta = 6.0$ ($\beta = 6.2$). Within the statistical errors KLM-Wilson results do not exhibit any appreciable a-dependence. We also tested another KLM prescription [9], including the shift of the mass M_P, obtaining results indistinguishable, within the errors, from the KLM-Wilson ones reported in table 1.

In order to obtain f_{B_s} and f_B, an extrapolation in the heavy quark mass well outside the range available in our numerical simulations is needed. Discretization errors can affect the final results in two ways. Not only do they change the actual values of the decay constants, but also deform the dependence of f_P on m_H.

In fig. 1, we show the Wilson and Clover re-

sults for $f_P/f_\pi\sqrt{M_P/\sigma^{1/2}}$ as a function of the dimensionless scale $\sigma^{1/2}/M_P$, with Wilson data uncorrected (above) and corrected (below) by the KLM prescription: the improvement is evident.
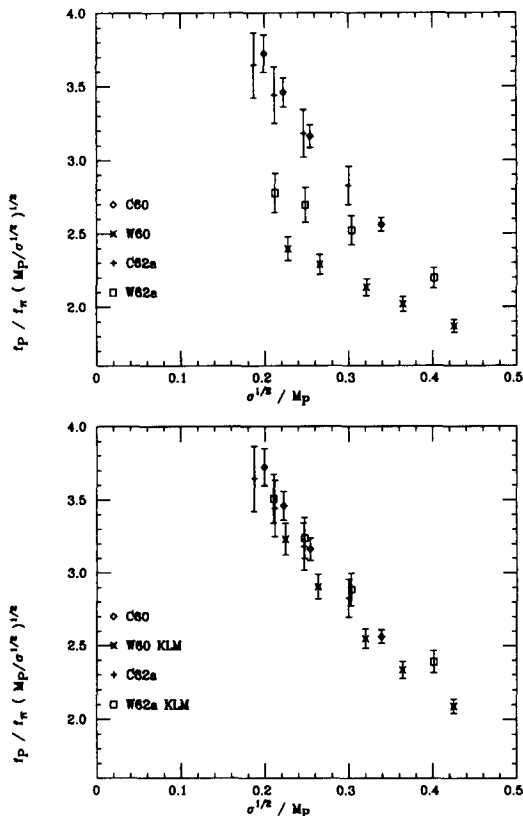

Figure 1. *Dependence of $f_P/f_\pi(M_P/\sigma^{1/2})^{1/2}$ on $\sigma^{1/2}/M_P$. For these points a linear extrapolation in the light quark masses to the chiral limit has been used.*

3. CONCLUSIONS

It is clear that, in spite of the very good accuracy of our data, any attempt to extrapolate our results to $a = 0$ in order to reduce discretization error would be fruitless: the results of the extrapolation are extremely sensitive to the choice of the scale, given the small range in a at dis-

posal. Thus we believe that the best estimate of the D- and B-meson decay constants is obtained from the Clover data at $\beta = 6.2$, by using the method of the eq. (2) (from a linear fit in the light quark masses, a quadratic fit in $1/M_P$ and without any KLM factor). By assuming quite conservative discretization errors we found

$$f_{D_s} = 237 \pm 16 \text{ MeV}, \quad f_D = 221 \pm 17 \text{ MeV},$$
$$f_{B_s} = 205 \pm 35 \text{ MeV}, \quad f_B = 180 \pm 32 \text{ MeV},$$
$$f_{D_s}/f_D = 1.07 \pm 0.04, \quad f_{B_s}/f_B = 1.14 \pm 0.08 \,.$$

in good agreement with previous estimates [10].

Further studies, with comparable (or smaller) statistical errors and physical volume, at smaller values of the lattice spacing, corresponding to $\beta = 6.4$ and 6.6, are required to reduce the $O(a)$ dependence of the decay constants. The use of the action of ref. [4] can be of great help in this respect.

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