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Illusory Figures: From Logic to Phenomenology

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Gestalt and cognitive-Bayesian approaches considered incompleteness as a necessary and sufficient factor for illusory figure formation. In this work, the role of incompleteness has been explored in terms of its inner logic and through an accurate phenomenal analysis of counterexamples and limiting or critical conditions. They demonstrated a bunch of logical issues and paradoxes, and, more importantly, that incompleteness is neither sufficient nor necessary to induce illusory figures. These issues are strongly reduced and possibly solved when the incompleteness is replaced by more simple concepts concerning interacting boundaries, grouping and surface filling-in processes during figure-ground segregation.

Keywords: illusory figures, visual illusion, Helmholtz's likelihood principle, simplicity principle, Bayesian decision theory

On Illusory Contours

The best-known example of illusory figures is Kanizsa's triangle (see Figure 1-left; Kanizsa., 1955, 1979; see also Schumann, 1900). Here, brightness enhancement and illusory contours are seen in the absence of luminance or chromatic changes across the perceived contours. Under these conditions, three black sectors and three angles, arranged respectively along vertexes and sides of a virtual triangle, are perceived as three black disks and an outlined triangle placed in depth behind a triangle with sharp boundary contours and brighter than the white background.

Similarly, the illusory bright diamond perceived in Figure 1-right emerges according to the arrangement of four groups of concentric outlined circular sectors placed on the corners of a diamond shape (see Ehrenstein, 1941).

Illusory contours have been explained on the basis of different approaches. In the next sections, we restrict our attention in greater detail to Gestalt theory, as mostly adapted by Kanizsa, and to Helmholtz's likelihood, as interpreted respectively by Gregory's cognitive hypothesis and Bayesian inference.

Gestalt Theory, Simplicity Principle, and Illusory Contours

Based on Gestalt theory, Kanizsa (1955, 1979) proposed that the necessary condition for the formation of the illusory figure is the presence of incompletenesses, or open figures, inducing amodal completion and processes of closure that in their turn elicit complete perceptual elements, partially occluded by an illusory object (i.e., the triangle or diamond as in Figure 1).

In order to account for the illusion, Kanizsa used terms, which are phenomenal outcomes, elements, or attributes that, even though they are clearly visible, are affected by some epistemological weaknesses. These terms are: incompleteness, open figure, amodal completion, and closure. The main issue, related to all of them and, more particularly, to the notion of

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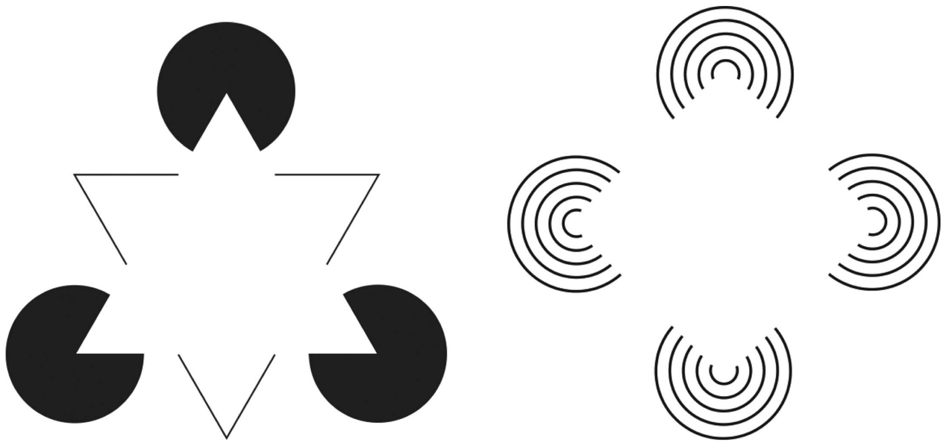
Figure 1
Kanizsa's Triangle (Left) and Ehrenstein's Diamond (Right)



"incompleteness," is the fact that they are complex phenomenal attributes, immediately visible (*Vorstellungen*, Metzger, 1975), used to explain the illusory contours but, at the same time, requiring to be explained in their turn.

The approach, adopted by Kanizsa, might suggest a sort of science of percepts *iuxta propria principia* (Bozzi, 1989; Kanizsa, 1980, 1991; Metzger, 1941, 1963, 1975) where the *explanans*, that is, the explanatory elements, and the *explanandum*, that is, the phenomenal target that has to be explained, belong to the same domain. Within this approach, the main risk is an epistemological circularity that is a logical fallacy in which the first element is used to explain the second and the second to explain the first (A is true because B is true; B is true because A is true). It is easy to take this risk by playing within a science of percepts *iuxta propria principia*, therefore while incompleteness explains the illusion, the explanation of incompleteness requires the illusion. In short, the illusion and incompleteness are mutually explained. Kanizsa's description does not necessarily imply this. A hierarchy, where open figures, closure, completion are primitives while illusory contours are secondary, might solve the epistemological circularity. Within these terms, there is no necessary intrinsic conflict between phenomenal and hierarchical. This circularity will be reconsidered more in details, both logically and phenomenologically, in the next sections.

Kanizsa's approach is also based on the simplicity-*Prägnanz* principle, regarded as phenomenal and as a brain process, according to which the visual system, like every physical system (Köhler, 1920), is considered as aimed at finding the simplest and the most stable organization consistent with the sensory input (Koffka, 1935). This principle expresses the idea that it is the stability of a whole that determines if its parts are perceived as parts. This idea was apparently overshadowed by the rise of single-cell recording, but it returned recently to the mainstream of cognitive neuroscience (see Ehrenstein et al., 2002; Epstein & Hatfield, 1994; Pomerantz & Kubovy, 1986; Wagemans et al., 2012a, 2012b).

Phenomenal simplicity refers to the belief that the visual system tends to capture a maximum of regularity. This is considered evolutionarily significant since regularities are likely to reflect meaningful invariants in the external world. Moreover, descriptive simplicity is also related to the assumption that the visual system tends to minimize the information load of mental representations implying an efficient use of internal resources. In terms of more recent computational approaches, the visual system chooses the simplest interpretation, the one defined by the least amount of information in terms of descriptive parameters due to regularities (Attneave, 1954; Hochberg & Mcalister, 1953), namely, the preferred perceptual organization is the one which elicits the briefest possible perceptual encoding (see also Atick & Redlich, 1990; Barlow et al., 1989; Blakemore, 1990). Therefore, the maximization of the explanatory power is equal to maximizing the simplicity of the encoding of the stimulus.

This suggests that a visual result or a description is meaningful if it carries information about the regularities of the stimulus, that is by reflecting the organization of elements and by specifying the structure of the stimulus. On the contrary, a code is meaningless if it is arbitrarily assigned to elements-strings, thus discounting the organization within the stimulus (Attneave & Frost, 1969; Hochberg & Mcalister, 1953; Koffka, 1935; Köhler, 1920; Leeuwenberg, 1969, 1971; Leeuwenberg & Boselie, 1988; Restle, 1970, 1979; Simon, 1972; Simon & Kotovsky, 1963; Vitz & Todd, 1969). Thus, the simplicity principle can be briefly described as follows: among the multiplicity of distal organizations that might account for a proximal stimulus, the one with the simplest code is predicted to be perceived and this reflects meaningful invariants and objects in the external world. The description of the illusory contours of Figure 1, as accounted by Kanizsa, is in line with this approach.

Starting from Gestalt approach, to model this simplicity principle several theories adopted descriptive coding languages, like for example the minimal model theory (Feldman, 1997, 2003, 2009) and complexity metrics (e.g., theory of Kolmogorov complexity, information theory, and structural information theory; see Chaitin, 1969; Chater, 1996; Kolmogorov, 1965; Leeuwenberg, 1969, 1971; Li & Vitànyi, 1997; Simon, 1972; Solomonoff, 1964a, 1964b). Within these views, the definition of information load (or complexity) is the number of different items extracted in order to specify or reproduce a given pattern (cfr. Leeuwenberg, 1969, 1971). It entails that the visual system selects the simplest interpretation of a given stimulus. More generally, the complexity of an interpretation is defined by the minimum information load (see also Attneave, 1954, for an early theoretical introduction on this idea).

Within these approaches, simplicity "minimizing coding length" is different from "maximizing regularity" (which may involve complex codes) and "finding the most stable organization consistent with the sensory input" because minimal codes may be fleeting, not stable. Stability implies building a fixed representation of everything at once, and that is likely to be complex.

In the field of statistical reasoning and inference, Occam's razor forms the foundation for the principle of minimum description length (Grünwald, 2000; Grünwald & Grünwald, 2007). In addition, it is automatically accommodated through Bayes factor model comparisons (e.g., Jeffreys, 1961; Jefferys & Berger, 1992; MacKay, 2003). Both minimum description length and Bayes factors feature prominently as principled methods to quantify the trade-off between parsimony and goodness of fit. Note that parsimony plays a role even in classical null-hypothesis significance testing, where the simplest null hypothesis is retained unless the data provide sufficient grounds for its rejection.

Various ways of quantifying the goodness of fit exist. One common expression involves a Euclidean distance metric between the data and the model's best prediction. Another measure involves the likelihood function, which expresses the likelihood of observing the data under the model, and it is maximized by the best fitting parameter estimates (Myung et al., 2000). The goodness of fit must be balanced against model complexity in order to avoid overfitting—that is, to avoid building models that well explain the data at hand, but fail in out-of-sample predictions. The principle of parsimony forces researchers to abandon complex models that are tweaked to the observed data in favor of simpler models that can generalize to new data sets.

What is basic within the simplicity principle as adapted to explain illusory figures is the phenomenal notion of incompleteness. Incompleteness is the benchmark and focal visual object around which revolves the formation of illusory objects.

On Helmholtz's Likelihood Principle and Unconscious Inference

A further important approach, based on top-down cognitive hypotheses, was aimed at solving the incompletenesses, also referred as "gaps," within the circular sectors and the missing components of the outlined triangle.

Gregory (1972, 1980, 1987) proposed that visual objects are similar to perceptual hypotheses, postulated to explain the unlikely gaps within stimulus patterns, according to which what is perceived is the object that under normal conditions would be most likely to produce the sensory stimulation. In other words, objects are like "unconscious inferences," that is, the results of inductive conclusions of a reasoning as used in the formation of scientific hypotheses. Therefore, in Figure 1, the illusory contours are perceived because the other obvious possibilities, that is, the fragments, abutting the occluding illusory shapes, would require a coincidental and unlikely arrangement.

Accordingly, Rock (1983, 1987) suggested that fragments similar to familiar objects trigger the cognitive hypothesis that an occluding surface is overlapped to complete figures. He proposed that symmetry, incompleteness, interruptions, gaps, alignments, familiarity, expectations, and general knowledge are cues triggering the supposed cognitive problem-solving process. This cognitive hypothesis subsumes the avoidance-of-coincidences principle, stating that the visual system tends to discharge perceptual interpretations induced by unlikely coincidences (Rock, 1983).

Along the same cognitive approach, Coren (1972) considered the incompleteness as a strong depth cue enabling the occluding triangle hypothesis. More in details, figural incompleteness triggers the emergence of an illusory figure superimposed on the gaps of the inducers, which amodally complete or continue behind the illusory figure, therefore suggesting that the plane of the illusion is above the one of the inducers.

These approaches are based on Helmholtz's likelihood principle (Von Helmholtz, 1867), whereby the sensory input is organized into the most probable distal object or event consistent with the sensory data (the proximal stimulus). This principle chooses the most likely interpretation and assumes that the visual system is highly veridical in terms of the external world.

From an evolutionary point of view, the rationale behind this principle is the need for a visual system to achieve likely percepts of the world. It is important that the representation and the perception fit; and this fitting should be fast and reliable. In fact, if the visual system was not likely, it would probably not have survived during the evolution. In this sense, the likelihood corresponds to the conditional probability of the distal pattern given the sensory input.

Unfortunately, it is not clear how this could be verified and how vision scientists might determine objective probabilities of real categories of distal scenes (cfr. Hoffman, 1998). Nevertheless, the likelihood principle and, within it, the Bayesian approach, generated several interesting and popular solutions related to how the visual system actually determines the relative likelihood of different candidate interpretations (how to determine what is most likely) and to how such principle translates into computational procedures.

On the other hand, the simplicity principle does not experience these problems, because it does not aim specifically at veridicality. The simplicity and the likelihood principle are two competing theories (see Hatfield & Epstein, 1985; Leeuwenberg & Boselie., 1988; Pomerantz & Kubovy, 1986; Rock, 1983) of perceptual organization and visual coding, which are difficult to settle because neither of the key elements were clearly defined. The general difference between the two is related to the fact that the visual system, in the case of the simplicity, obeys a more general principle of economy, while in the case of the likelihood, it obeys a general principle of probability.

Nevertheless, these two terms might be only apparently different or may be considered as two sides or two different ways of considering the same visual process. Mach (1914, 1959) suggested that vision acts in conformity with the principle of economy, and, at the same time, in conformity with the principle of probability. Chater (1996) demonstrated mathematically that these key elements can be unified and considered equivalent within the theory of Kolmogorov complexity (Chaitin, 1969; Kolmogorov, 1965; Li & Vitànyi, 1997; Rissanen, 1978; Solomonoff, 1964a, 1964b). Kolmogorov (1965) defined the algorithmic and descriptive complexity of an object to be the length of the shortest binary computer program that describes the object.

Feldman (1997, 2003, 2009) presented a simplicity approach, called minimal model theory, and, in agreement with Chater (1996), suggested that the visual interpretation, whose description is of minimum length, is the one that most likely is also the most veridical. Usually, the tendency of choosing a visual object that minimizes the description length is the same as the tendency of choosing a hypothesis that maximizes the likelihood. In brief, the most likely hypothesis about perceptual organization is, at the same time, the objects supporting the shortest description of the stimulus.

As for the simplicity principle, within Helmholtz's likelihood the phenomenal notion of incompleteness (gap, missing component) represents a true benchmark and focal term in order to solve the formation of illusory objects. As a matter of fact, a circle with a missing sector or with a gap is a very unlikely occurrence and the likelihood that all such objects with missing parts are casually aligned to create a triangle is very weak. This point will be definitely more elaborated in the next section.

From Helmholtz's Likelihood Principle to Bayesian Decision Theory

Strongly related to Helmholtz's likelihood, Bayesian statistical decision theory is a principled method of optimal reasoning under uncertainty. It is the best math tool to formalize Helmholtz's and Gregory's notions of unconscious inference. Bayes' rule is given by:

$$p(H|D) = \frac{p(D|H)p(H)}{p(D)} \tag{1}$$

According to Bayes' theorem, for data D, the posterior probability $p(H|D)$ of hypothesis H (how likely H is for a given D) is proportional to the product of the prior probability $p(\mathrm{H})$ that H occurs, that is, the probability that interpretation H occurs independently of proximal stimulus D (how likely H is in itself), and the likelihood $p(D|H)$ that D occurs if H were true, that is, the probability that proximal stimulus D occurs if interpretation H were true (how likely D is under H). The probability $p(D)$ that D occurs is the normalization factor.

Briefly, the Bayesian approach aims to calculate the posterior probability distribution over the hypotheses and to select the most likely hypothesis with the highest posterior probability under the prior and conditional probabilities. The normalization factor can be omitted. The prior denotes how good an interpretation is independently of the proximal stimulus, and the conditional denotes how good the proximal stimulus is if the interpretation were true.

By applying the previous equation to perceptual organization, prior probability distributions $p(H)$ could represent the knowledge of the regularities of possible object shapes, while the likelihood distributions $p(D|H)$ could represent the knowledge of how objects are created through projection onto the retina.

Simple elements, like those illustrated in Figure 1, are consistent with many different physical shapes, depending on viewpoint, and are described by the "likelihood function." Nevertheless, some are more prevalent in normal viewing than others, leading to a probability distribution referred to as the "prior." The prior is combined with the likelihood to yield the "posterior probability distribution," narrower than either the prior or the likelihood, whose maximum is taken as the statistically best estimate of the shape creating that image. If the prior is appropriate, the Bayesian framework provides the most efficient method to infer a 3D shape corresponding to simple 2D line drawing.

This approach can effectively model visual illusions like Kanizsa's triangle and Ehrenstein illusion. In short, for both phenomena, the gaps are most likely due to a closer occluding surface, seen even if it is not physically present as a stimulus. The visual system creates a surface that, although illusory, is likely true. Assuming that the sensory system is Bayesian, Kanizsa's triangle is accounted for by considering that a circle with a missing sector is a very unlikely occurrence. The prior for these circular sectors is that they are whole circles. Furthermore, the likelihood that all such objects with missing components are casually aligned to create the shape of a triangle is very small. In short, Kanizsa's illusion can be explained in a Bayesian framework if it is assumed that perfect circles and completed triangles are more probable in Nature than are other possibilities. Therefore, within a Bayesian approach, the occluding triangle is the hypothesis with the greatest probability of occurrence.

As first demonstrated by the Gestalt psychologists, perception usually converges on a single phenomenal outcome that corresponds to the current estimate of reality (see Kanizsa, 1979). Bayesian procedures do not yield a single conclusion, but they rather yield a full posterior distribution that assigns a degree of belief to every hypothesis in the pattern of stimuli. When a single conclusion is required, what is considered is the maximum a posteriori interpretation that represents the best Bayesian guess of the shape to produce the image.

Kanizsa's triangle is equally consistent with a continuous white triangle superimposed over three regular black circles and with three unlikely circular sectors arranged symmetrically to face each other. However, both the process of amodal completion and the natural statistics of the world make the illusory triangle more probable and the coincidence of the sectors arrangement very unlikely. This interpretation is in line with Bayes' inference. As a matter of fact, the likelihood that all such sectors are casually aligned to create the shape of a triangle is very small if compared with the occluding triangle whose probability of occurrence is much greater. Moreover, the prior for these circular sectors is that they are complete circles.

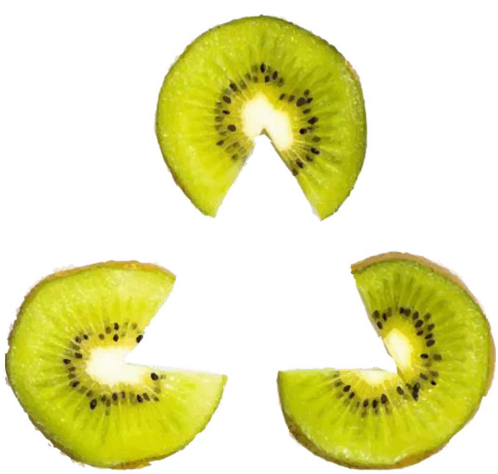
The two illusory figures of Figure 1 clearly corroborate the likelihood principle and the unconscious inferences as also related to Bayes' rule, though, in the next sections, we will logically and phenomenologically demonstrate that incompleteness and other cues used in Kanizsa's Gestalt approach and for the cognitive hypothesis are neither necessary nor sufficient factors in inducing illusory figures (cfr. Pinna & Grossberg, 2006).

Further Examples of Illusory Figures

In Figure 2, three slices of kiwi induce an ambiguous Kanizsa's triangle. Phenomenally, the three slices can be both perceived as sectors and as circular slices. It is worth to note that even when they are perceived as sectors, they induce an illusory triangle not necessarily occluding circular slices. In other words, the sectors do not necessarily become complete slices as a consequence of the formation of the triangle. The incompleteness of the kiwi illusory triangle is not completed but remains incomplete though inducing an illusory figure.

A second remark, useful for our purposes is related to the fact that, under these new conditions, the likelihood that all such sectors are casually aligned to create the shape of a triangle is now very high. This condition pushes forward the need to explore the phenomenal incompleteness more in depth, since it is the key element to test Bayes' inference and the related approaches previously described as appropriate interpretations of illusory contours.

Figure 2
Kiwi's Illusory Triangle



Note. See the online article for the color version of this figure.

A further intriguing example is illustrated in Figure 3. Here, complete elements (real ancient little statues) appear as placed along the circumference of an illusory circle, which is not occluding any component or part of the inducers. The inner illusory circle does not appear as clear and distinct as the ones of Figure 1. More effective outcomes with sharp illusory contours will be presented in the next sections. Nevertheless, this pattern weakens the notion of incompleteness as a primary factor of illusory figures and push to reconsider it in a new more general and simple meaning.

As a matter of fact, phenomenal incompleteness represents a significant issue not only to explain illusory figures but also to cast light on the more general problems of organization of the visual world and object segregation. Within the problem of organization, illusory figures are usually considered as basic tools to understand how and why the visual system segregates or creates objects with a particular shape, color, and depth stratification. As a matter of fact, they subsume and clearly show all these integrated attributes mutually bounded and useful to be accounted for in a simple unique way, as previously suggested. Understanding the role played by incompleteness in inducing illusory figures can thus be useful for understanding the principles of organization (the How) of perceptual forms and the more general logic of perception (the Why). To this purpose, incompleteness will be studied in the next sections through its inner principles and inner logic.

The role of incompleteness is analyzed in the next three sections by: (a) describing and arguing the inner logic subtended by the use of the term "incompleteness," (b) presenting new conditions, whose purpose is to clarify the phenomenology of incompleteness in terms of necessary and sufficient condition, and (c) suggesting an alternative hypothesis to explain illusory figures on the basis of the emerged phenomenological evidences. The next sections are based on a previous work (Pinna & Grossberg, 2006), which is here considered as the central point for the resulting arguments and counterexamples proposed.

Figure 3
Illusory Circle With Complete Inducers

Note. See the online article for the color version of this figure.

Logic and Paradox of Incompleteness

Within the previous theoretical views, incompleteness can be considered both as a geometrical/structural factor inducing illusory contours and, at the same time, as a perceptual property, beforehand, perceived as such and, afterward, activating object hypotheses, unconscious Bayesian inferences, or completion processes.

From a logical point of view, if incompleteness is assumed as a geometrical factor, then it cannot be defined as incompleteness, because the term "incompleteness" implies a perceptual phenomenon and not a geometrical property (Koffka, 1935). Consequently, this use of the term "incompleteness" leads to the experience error (Köhler, 1947), that is, the implicit naive belief that the structure of perceptual experience coincides with the optical array. Moreover, if considered as a geometrical property, incompleteness does not require any solution, explanation, or completion.

Nevertheless, by assuming that incompleteness, open figures, or irregular elements can activate neural dynamics toward amodal completion and closure, they are expected to always reveal two states: the preceding, uncompleted and open, and the succeeding, completed, and closed state. However, since only one state is perceived, this entails that, as a geometrical property, incompleteness cannot be a cause of itself considered as a perceptual phenomenon that in its turn creates an illusory figure.

On the other hand, if incompleteness is assumed as a perceptual property, it entails that, as such, it cannot be assumed as the cause of another perceptual attribute placed at the same epistemological level. Moreover, from an empirical perspective, if incompleteness is perceived as incompleteness, there is no need to be discounted or completed or considered as the cause of illusory figures. This is especially true given that incompleteness is in most conditions a perceptual property autonomous or independent from illusory figures. Furthermore, it cannot be considered as such, since, as soon as it is perceived, it is unperceived. This is a clear *reductio ad absurdum*. It follows that incompleteness cannot be perceived because, once it is seen, it should be immediately completed and explained. It is first perceived as incomplete and then completed, but to be completed it should first be perceived as incomplete.

This is a clear circular argument, that is, a logical antinomy, namely, it is perceived if and only if it is not perceived, or it is not perceived if and only if it is perceived. This is the paradox of incompleteness. The paradox can also be stated as follows: To perceive an incompleteness that has to be explained and completed, first, it should become a conscious phenomenal property and, then, it should become unconscious so that its perceptual result is annulled or completed. In other words, the incompleteness is not perceived and, thus, completed, if and only if it is perceived and, hence, not completed.

Furthermore, the assumption that incompleteness is a depth cue implies that, as with all other depth cues, it should be perceived, but if it is perceived it should not be perceived; therefore, the logic argument is the same as before.

Finally, if incompleteness is a perceptual property, it is not a perceptual property. In other words, incompleteness can never be perceived because it should be always completed. Said yet another way, incompleteness should never be perceived even under conditions independent from illusory figures.

The previous arguments reject incompleteness as a factor for explaining illusory figures on the basis of mere logical reasoning. The completion preceding the inducers should be rejected. In the next sections, the role of incompleteness is further weakened and reconsidered through new phenomena.

The main purpose of the following sections is to demonstrate phenomenologically that incompleteness is neither necessary nor sufficient to explain illusory figures. The demonstration of this statement follows four steps: (a) incompleteness is not sufficient, (b) illusory figures do not necessarily complete incompletenesses, (c) the shape of incompleteness does not predict the shape of illusory figures, and (d) incompleteness is not necessary.

The phenomenology of incompleteness will be finally accomplished by demonstrating that Gestalt principles of grouping and figure-ground segregation of discontinuities within the boundary contours and brightness inhomogeneities could be responsible for illusory figures, without any need of invoking incompleteness.

Incompleteness is Not a Sufficient Condition

The first argument assumes that incompleteness is not a sufficient condition in inducing illusory figures. A condition A (in our case, incompleteness) is said to be sufficient for a condition B (illusory figure), if and only if the truth (namely, existence or occurrence) of A brings about the truth (existence or occurrence) of B. In short, A cannot occur without B, or whenever A occurs, B occurs.

Although the necessary logical condition cannot be easily refuted, that is, that all illusory figures contain incomplete inducing elements or that complete inducers cannot produce illusory figures, it can be demonstrated more easily that not all occurrences of incomplete elements produce illusory figures (the sufficient condition). In terms of the approaches we are testing, the sufficient condition is less interesting than the necessary one, but it is nonetheless significant to understand the phenomenal properties of incompletenesses and of the perceptual organization involved.

Figure 4a–b shows an example of phenomenal incomplete circle and diamond not inducing any illusory figure. They can be described as incomplete figures with missing parts inward (concave discontinuities) on the right-side and on the corner of the diamond. The term "incomplete" is phenomenal and general, but more specifically described as gnawed or nibbled objects.

Within an epistemological and logical perspective, a clearer distinction between the term "incompleteness" and a geometrical term like "concave discontinuities" of the contours is needed. As a matter of fact, the geometrical property of concave discontinuities is not subjected to the paradox of incompleteness. At the same time, the "concave discontinuities" can also manifest a phenomenal meaning. They are in fact perceptual outcomes, phenomenally not requiring any completion. They do not tend to any further perceptual state. Therefore, epistemologically and phenomenologically, the geometrical concave discontinuities do not induce the perception of incomplete objects.

By introducing "concave discontinuities," our purpose is not to define incompleteness but to highlight the role played by discontinuities along the boundaries, useful to understand incompletenesses, and more clearly related to principles of grouping and figure-ground segregation.

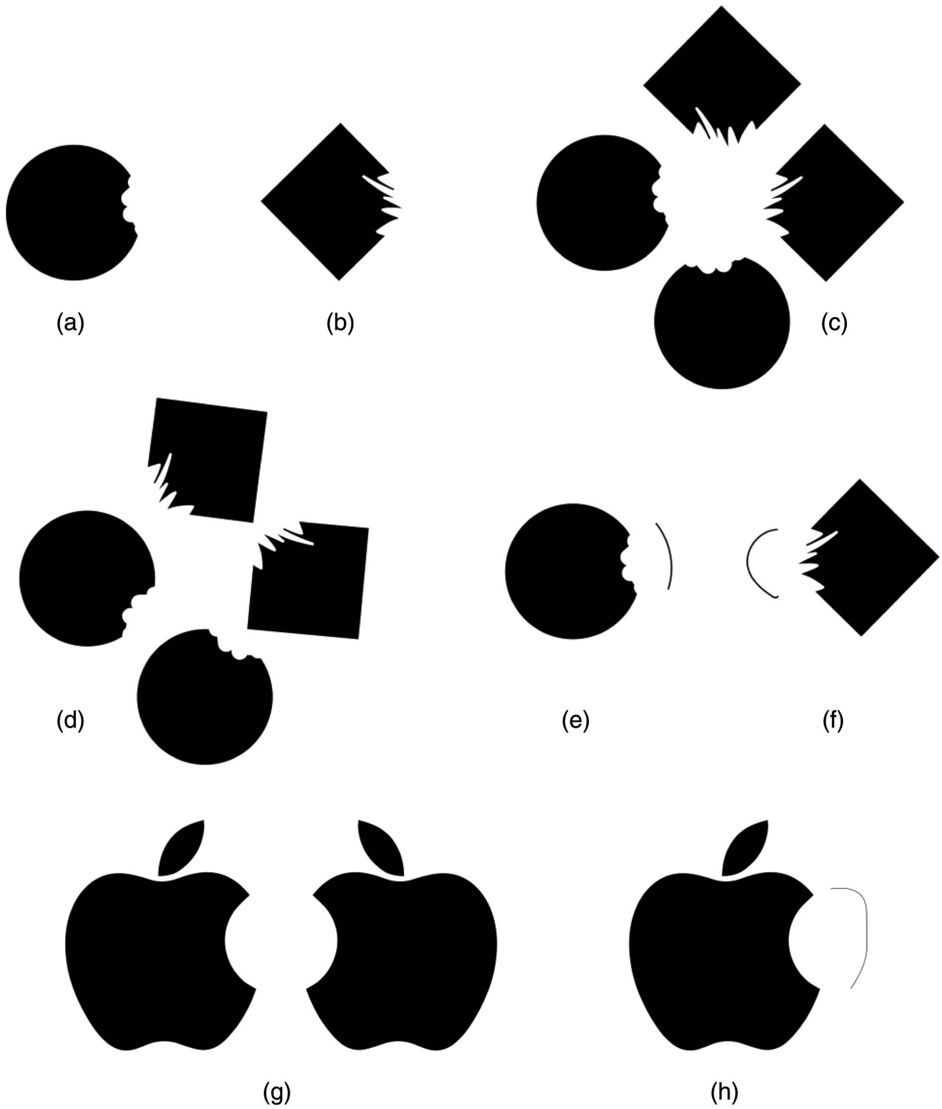
It follows that, if a region with concave discontinuities segregates from the other boundaries of the same object, due to the similarity/dissimilarity Gestalt principle, then they can group with similar concave discontinuous boundaries belonging to other objects, as shown in the next figures. The replacement of incompletenesses with "concave discontinuities" elicits the dynamics of grouping and figure-ground segregation and allows to solve the logical and phenomenological argument against "incompleteness." Furthermore, it suggests hypotheses not restricted to illusory figures but more related to segmentation and to the organization processes of vision. This alternative acceptation of incompleteness will be clarified below.

Concerning the description of Figure 4a–b, it can be objected that this incompleteness does not induce any illusory figure since it does not need to be completed. However, if this is true, we have a clear demonstration that incompleteness is not a sufficient factor. If incompleteness is not sufficient, then the question is: under which geometrical conditions does incompleteness get completed, as suggested by Gregory, Rock, Coren, and Kanizsa? A possible answer to the question is illustrated in Figure 4c, where, by favoring (facing) the grouping of discontinuities of the circle and the diamond, an irregular illusory amoeboid figure partially occluding a circle and a diamond is perceived.

These outcomes suggest that the classical laws of organization, as described by Wertheimer (1923),

Figure 4

(a–f) Incompleteness Is Not Sufficient—(a, b) Incomplete Circle and Diamond Without Illusory Figure; (c) Illusory Amoeboid Figure Partially Occluding Four Objects—Two Circles and Two Diamonds; (d) By Breaking the Good Continuation the Illusory Figure Largely Disappears; (e, f) Illusory Figures Partially Occluding a Circle and a Diamond (g–h) Incompleteness Is Not Sufficient



play a basic role. Incompletenesses or concave discontinuities group on the base of proximity, closure, similarity of contours, and good continuation factors, creating illusory contours. A corollary suggests that, by breaking their good continuation (cfr. Figure 4d), the illusory object largely disappears. The roles of the closure factor and articulation-without-rests are shown in Figure 4e–f, where the arcs represent the rests included within the whole organization, assuming the role of the boundaries of the illusory figure partially occluding the black circle and the diamond.

Further and more effective examples, demonstrating that incompleteness is not sufficient and in favor of the role of closure and articulation-without-rests factors, are illustrated in Figure 4g–h.

Figure 5-left can be described as an incomplete black square, whose fragments appear like erasures or as white paste, painted with a tempera in the inner edges of the square.

Under these conditions, very weak or no illusory contours are perceived outside the square. The incompleteness here assumes a specific meaning: erasure or painting. Phenomenologically, this is not a mere incompleteness but "something" likely more complex. This entails that incompleteness is an undetermined property that can be resolved into a large number of meanings, where just one of them is related to the formation of illusory contours. In Figure 5-right, the illusory figures appear as non-homogeneous paintings or white paste across and along the circle components. This multiplicity of meanings and uncertainty weakens more and more the phenomenological role played by incompleteness as assumed within the previous theories. They suggest a notion of incompleteness as strongly related to principles of Gestalt grouping and to meaning assignment to contours discontinuities.

In Figure 6a, the word ART is perceived (Pinna, 1990). The similarity/dissimilarity of the kind of the boundaries composing the R letter as well as the grouping of concave discontinuities, due to the good continuation principle, can be considered as responsible for the perception of the word ART. In addition, the surroundedness principle of figure-ground segregation (Rubin, 1915, 1921) and past experience (Wertheimer, 1923) likely influence the pop out of the R letter. The illusory R appears in front of A and T, which complete amodally.

The role of past experience is more salient in Figure 6b, where the role of similarity/dissimilarity principle is weakened and the incompleteness within the T groups with the empty space between A and T, which, due to the surroundedness principle, tends to appear as a figure. Surroundedness operates also with the R considered as the incompleteness of T, which in its turn can be seen as a concave discontinuity, that is, a special case of the concave grouping principle (Wertheimer, 1923). It is worth noting that the R appears on the same plane of the A, but it is partially occluding the T.

In Figure 6c, the word ART emerges less strongly than in Figure 6b. The presence of a line separating the component of the R within the T and the empty space between the A and the T weaken the grouping of the R and favor the amodal completion of the former component behind the latter. The R does not appear as a clear illusory figure, although it can be clearly recognized.

Moreover, the separate component of the R is seen completing amodally behind the other component of the R, thus making the surrounding black region of the T to appear as an empty space, that is, similarly to a window open on a black background.

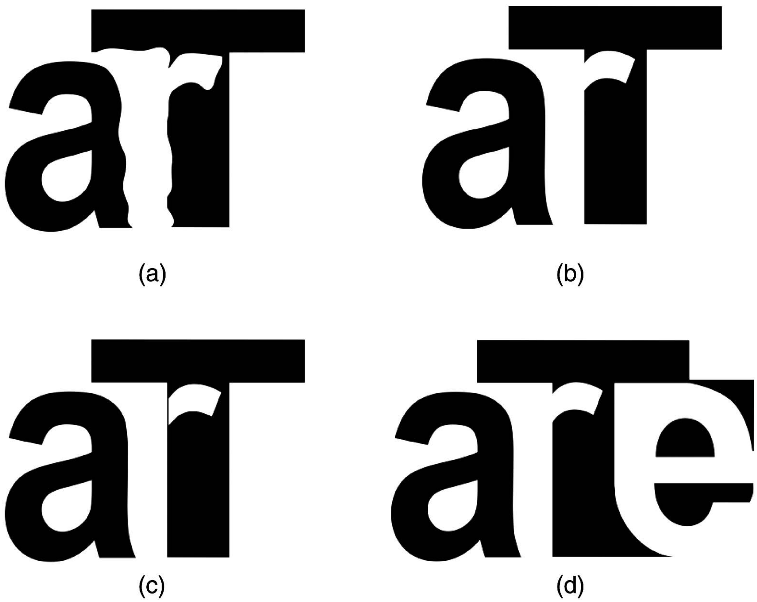
In Figure 6d, the word ARTE (Italian for ART) is now read. The letter E is perceived despite the absence of incompletenesses or local concave discontinuities of the R. On the contrary, on its right side, the T presents convexities or additions. Both R and E pop out from the background as figures, acquiring clear figural qualities. The letter

Figure 5

Roles of Incompletenesses in Different Organizations: (Left) a White Paste in the Inner Edges of the Square and (Right) Illusory Contours and White Paste

Figure 6

Gestalt Grouping Principles as Alternatives to Incompleteness: (a) the Word ART; (b) the Illusory R Emerges Due to Surroundedness and Past Experience Principles; (c) the Line Separating the Component of the R Within the T Weakens the Emergence and Segregation of the Word ART; the R Does Not Appear as an Illusory Figure; (d) the Word ARTE: the E Letter Pops out Due to Past Experience, Surroundedness, Good Continuation, Convex Discontinuities Along the T Letter, Closure, and Articulation Without Rests



E emerges from a black (upper-right and lower-left corners) and white (upper-left and lower-right corners) square that is seen as a background. The E appears with curved boundaries. The perception of the E and its background demonstrates that the boundaries belong unilaterally to the illusory E. The E letter pops out as a figure likely because of the following principles: past experience, surroundedness, good continuation, convex discontinuities along the T letter, closure, and articulation-without-rests. Despite the fact that perception of the four alphabetical letters seems to be ruled by different figural principles, they group together making the reading of the word ARTE easy.

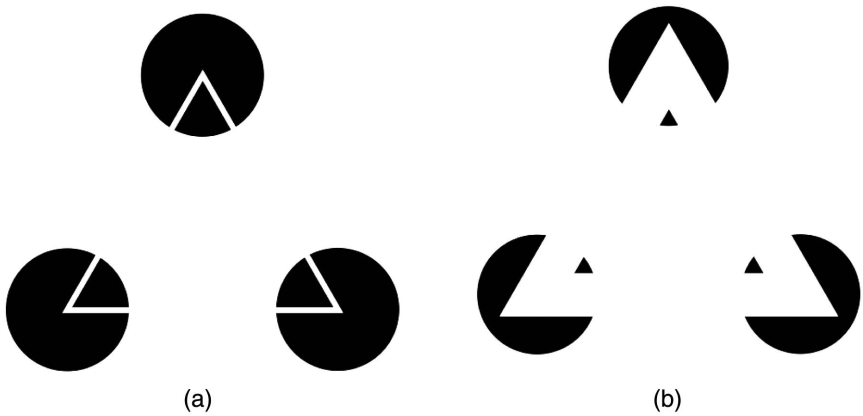
Because the convex discontinuities along the right side of the T are the opposite of the concave discontinuities on the left side, considered as the low-level description of the incompleteness, the illusory E letter is induced from something that is the opposite of incompleteness; namely, convexities and additions.

These outcomes suggest that grouping and figure-ground segregation principles play a true role in the explanation of illusory figures and, more specifically, in their perceptual organization.

In Figure 7a, the three corners of a virtual triangle do not induce an illusory triangle across the white space between the circles. Rather, the percept is more commonly a white perimeter (contours) of a triangle partially seen through three holes on a white surface showing a black background. The disks appear like black holes whose boundaries belong to the surrounding white surface, placed on a homogeneous black background. The white surface is then seen as punched and partially occluding a triangle. Again this result supports the basic role of figure-ground and grouping principles creating patterns with new emergent properties depending on the dynamics among the principles involved.

These results clearly demonstrate that, although incompletenesses are geometrically aligned to favor good continuation, and although closure, proximity,

Figure 7
A Case of the Proximity Factor Induces (a) a White Triangle Partially Perceived Through Three Circular Holes on a White Surface and (b) an Illusory Triangle



and *Prägnanz* principles work synergistically and the boundary contour of the triangle is thin enough to induce a strong figural effect due to surroundedness and proximity principles of figure-ground segregation (Rubin, 1921), the expected illusory triangle is absent. However, by increasing the width of the boundary contours, an illusory figure pops out, in spite of the less favorable conditions in terms of figure-ground segregation (Figure 7b).

The same argument occurs for Figure 8a, where the breaks along each ray are perceived as bright short dashes upon the rays that do not complete in an illusory circumference among the ray interspace, but appear to create a virtual circumference, that is, a circumference made of dots or dashes. Differently from Figure 7a, the rays are not seen as holes and the bright dashes do not completed amodally behind the white interspace, perceived as a background and not as a surface.

This outcome answers a possible counterargument related to Figure 7a. It states that, the fact that in Figure 7a an illusory triangle is not created depends on the reversed and complementary effect (disks as holes, illusory triangle as amodal triangle, etc.) induced as object hypotheses that can analogously explain the gaps within the stimulus pattern. In short, the amodal triangle is the most simple and likely solution to the incompleteness within the circular holes. This counterargument is stopped by the results of Figure 8a, that is a true counterexample for the counterargument since the illusory figure does not complete the bright interruptions, as does happen in Figure 8b.

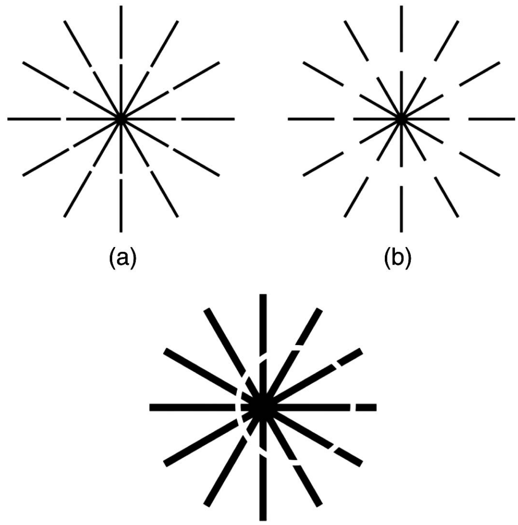
Without invoking incompleteness, there is a simpler factor eliciting the different percepts of Figures 7a–b and 8a–b. It is the ratio between the width of each interruption and the interspace amplitude among rays. It follows as a corollary that by increasing the width of the amodal triangle of Figure 7a as in Figure 7b, or of the virtual circumference of Figure 8a as in Figure 8 b, the ratio changes accordingly. A clear demonstration of the corollary is also perceived in Figure 8c, where the right side of the figure does not show any illusory circumference but just isolated or unconnected bright dashes, while in left side the dashes group in a clear illusory arc superimposed to both the rays and white background.

It is worthwhile to highlight that the phenomenal difference between left and right side of Figure 8c is so great to win against the Gestalt tendency of good continuation among dashes, synergistically enhanced by the illusory arc on the left side. The supposed ratio among width of the dashes and interspace among rays is a special condition of the proximity factor, pitting horizontal and vertical proximities one against the other (see also Pinna, 2011).

To conclude this section, in order to explain phenomenally the previous results, there is no need to invoke completion processes of incomplete inducing elements. They can be more simply explained in terms of grouping factors. This argument will be further deepened in the next section.

Figure 8

(a) A Virtual Circumference Made of Dots or Dashes, (b) an Illusory Annulus, and (c) Unconnected Bright Dashes on the Right Side and an Illusory Arc on the Left Side



Illusory Figures Do Not Complete Incompletenesses

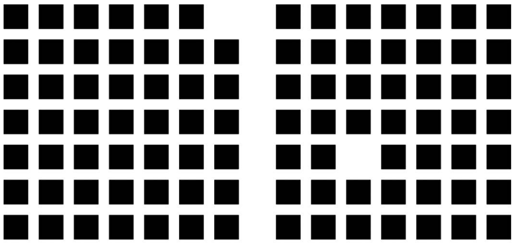
In Figure 9a, a set of squares, arranged in a square matrix, with a missing element in the left upper corner is perceived (cfr. Pinna, 1990). This is a case of "incompleteness" without illusory figure, that is, the not-sufficient condition. In Figure 9b, the matrix of squares appears again incomplete but with an illusory bright square, whose sizes are larger than the ones of the surrounding black squares. The perception of the total occlusion of a small square, due to the illusory large square, is not a clear phenomenal evidence but it is more similar to an inference. Furthermore, the four black squares, arranged in a cross and placed all around the illusory one, do not appear incomplete or partially occluded, although they are juxtaposed and create a T-junctions with the illusory square and pairwise-collinear, which is a basic constraint eliciting amodal completion.

It can be argued that, under these conditions, the perceived incompleteness must be global; that is, related to the grid and not to single squares. Even if it were so, the matrix does not appear completed. To appear completed, a small black square should be perceived behind the illusory large square, that does not appear occluding anything even if it is tangent to the sides of four black squares all around, which in their turn do not appear incomplete or partially occluded. Figure 9b is a case of incompleteness not completed by an illusory figure neither locally nor globally. This result represents a logical refutation of the role of incompleteness, whether incompleteness is assumed as a necessary condition.

A further counterargument, derived from the previous approaches based on incompleteness, states that, when the illusory figure is perceived, the inducing elements complete themselves amodally in a more global figure that is different from the geometrical inducing elements. If this is true, incompleteness is assumed as an a posteriori result defined and known only after the perception of the illusory figure. This argument implies the same paradoxes already defined. However, even if we accept the logic of the argument, it can be phenomenologically weakened by the outcomes of Figure 9b, and definitively rejected through more probing counterexamples shown below.

In Figure 10a, the ungrouped and separated square is expected to complete the square matrix of squares by assigning to the whole pattern the following meaning "a grid of squares with one

Figure 9
Incompletenesses Are Not Necessarily Completed by Illusory Figures: (a) a Square Matrix With a Missing Element in the Left Upper Corner and (b) an Incomplete Square Matrix With an Illusory Bright Square Larger Than the Black Squares and Not Occluding Any Element



moved up." This result is expected to be the only one, the simplest, the most likely, and the most veridical. There is no need to create further hypothesis and there is no need to "create" an illusory figure. Nevertheless, the separated square does not complete the missing element within the grid, but an illusory bright figure similar to a rectangular vertical stripe is perceived going from the lower square on the grid within the empty region and the bottom side of the ungrouped square. Again, the phenomenal notion of incompleteness manifests clear logical and phenomenological fallacies. "Incompleteness" appears totally inadequate in order to explain illusory contours.

These results are further corroborated by Figure 10b, where the illusory figure is enhanced in spite of the ungrouped square placed closer to the matrix and thus suggesting the cognitive hypothesis of a square moved up. The illusory contours persist also in Figure 10c, where the incompleteness, completed by squares moved up, are stronger and stronger, even by changing the color of the square. Playing with words, these outcomes suggest that "incompletenesses" is not completed by phenomenal shifting of elements.

The perceptual results of Figure 11 are different from those of Figure 9 in that incompletenesses are here accompanied by illusory figures similar to a white scribble in front of one entirely occluded (Figure 11a–b). The scribble perceived in Figure 11c, even if related to the incompleteness of the squares, appears to lie behind what appear as white bars of a window showing a dark inner room. This result is similar to the one described in Figure 7a. In Figure 11d, the main body of the scribble is white and the proximity ratio among distances is changed, so that phenomenally the inner body of the scribble appears in front of the partially occluded white bars while its legs appear as placed behind the bars. Therefore, the bars are both occluded and occluding the sort of white intertwined insect.

The question is then: which are the inducing elements of the illusory insect, the white bars or the black squares? If we assume incompleteness as a necessary factor, then the question requires the following paradoxical answer: the effect is cause of its cause. In other terms, the incompleteness of the bars and not the one of the squares can be decided only a posteriori and it becomes the cause of the perception of the incompleteness of the bars and not of the squares. This is paradoxical only if incompleteness is considered a necessary factor. This criticism will be logically and phenomenologically developed in the next sections from an alternative point of view.

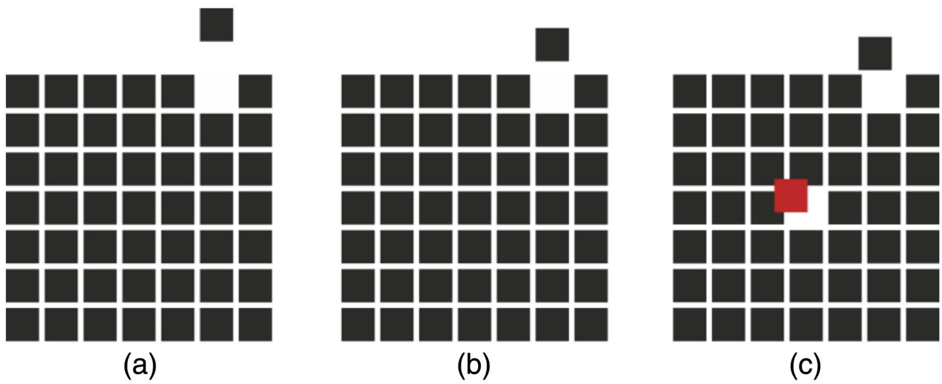
Incompleteness Does Not Predict the Shape of Illusory Figures

The questions are now: incompleteness of what? Once the shape of incompleteness has been defined, can the shape of the illusory figure be predicted? Does the shape of the illusory figure correspond to the shape of incompleteness? Moreover, does the shape of the illusory contours agree with the shape of real contours when they complete the shape of the same incompleteness?

A plethora of previous results showed the equivalence between illusory and real contours. Illusory contours have been demonstrated like real contours: in producing geometrical illusions (Farné, 1968; Bradley & Dumais, 1975; Bradley & Petry, 1977; Gregory, 1972; Meyer & Garges, 1979; Pastore, 1971); in kinetic depth information (Bradley & Lee, 1982); in apparent and stroboscopic motion (Ramachandran, 1985; Sigman & Rock, 1974; von Grünau, 1979); in figural after-effects (Meyer & Phillips, 1980; Smith & Over, 1976, 1979); used as targets or masks in visual masking experiments (Reynolds, 1981; Weisstein et al., 1974); in information-processing tasks as a landmark aiding the localization of elements in visual space (Pomerantz et al., 1981). von der Heydt et al. (1984), and Peterhans and von der Heydt (1987) found that

Figure 10

"Incompletenesses" Issues: (a) a Matrix of Squares With One Moved Up Plus a Rectangular Vertical Illusory Stripe and (b, c) Illusory Squares Persist in Spite of the Clear Perception of Squares Moved Up

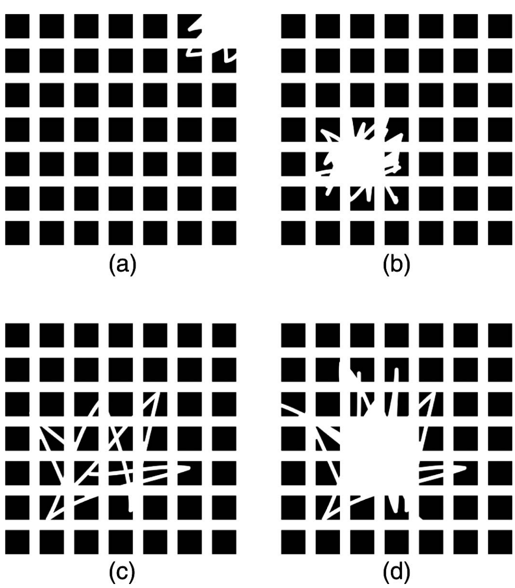


Note. See the online article for the color version of this figure.

neurons in cortical area V2 of macaques respond at locations where illusory contours are perceived. These neurons respond with a similar slightly delayed excitatory response to both illusory and real contours that crosses their receptive fields, and both real and illusory contours produced similar orientation tuning in these cells.

Figure 11

Different Roles of Incompletenesses: Scribbles of White Tint in Front of One Entirely Occluded (a, b) Square; (c) the Scribble Lies Behind the Crossed White Bars of a Window Showing the Dim Interior of a Room; and (d) the Inner Irregular White Shape, Similar to the Body of an Insect, Appears in Front of the Partially Occluded Crossed Bars While Its Legs Appear Intertwined in the Bars



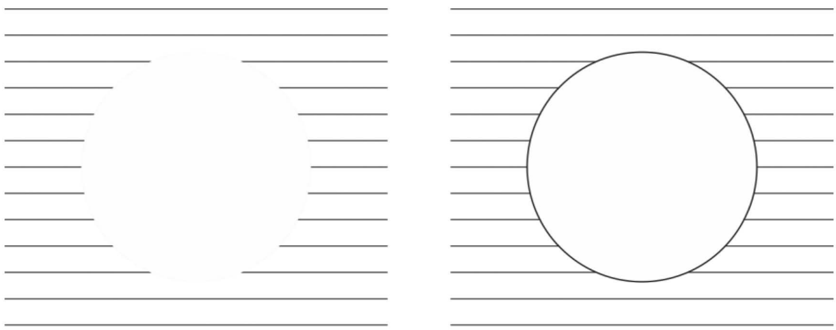
In spite of these results, Figure 12 refutes this hypothesis and highlights clear differences between the two kinds of contours. In Figure 12a, a parallel distribution of horizontal lines encloses a circular gap. However, it does not appear like an illusory circle but as an elongated or distorted shape similar to a lemon. The "true" circular shape of the incompletenesses does not correspond to the shapes of the illusory figures, as illustrated in Figure 12b.

The main point is: Why should a circular gap be completed as described if the simplest figure that could solve the problem of incompleteness is an illusory circle? The complexity of the solution is far from being considered as the simplest solution. If incompleteness is assumed to be a necessary factor, is it possible to predict the exact shape of the illusory figure aimed to complete the gap in the simplest way? Figure 12a–b demonstrate that the answer is negative, and that real and illusory contours do not necessarily match.

Figure 13 sets the inverse problem. The shapes of the illusory figures fit the gaps, however completing the gaps with real contours, the resulting shapes do not match the ones induced by the illusory figures.

In Figure 13a-left, an illusory disk is perceived filling the circular gap in the center of radial stripes. By replacing the illusory contours with real ones (Figure 13a-middle), the illusion of angularity is perceived (Figure 13a-right, Pinna, 1991),

Figure 12
The Illusory Figure (a) Differs From the Shape of the Incompleteness (b)



according to which the circle appears to be polygonal with blunt angles directed toward the inside of the black stripes. When the gap is a polygon with each vertex lying between two contiguous radial stripes, an illusory polygon, very close to the gap shape, appears (Figure 13b-left). But when it is replaced by real contours (Figure 13b-middle), the sides of the polygon look convex with swellings inside the stripes. The vertices appear much less pointed and slightly rounded off (Figure 13b-right). By viewing globally, the polygon appears more circular than the real circle illustrated in Figure 13a-right. If the gap is a polygon with vertices lying within the black stripes (Figure 13 c-left), an illusory polygon is seen. But, when it is replaced with true contours (Figure 13c-middle), the polygon appears more polygonal, pointed, or sharper with the polygon sides appearing concave and the vertices going even more inward into the stripes (Figure 13c-right). On the basis of these results, the question is: if the illusory figure is the solution to the problem created by the gaps, why does the phenomenal shape appear different by replacing illusory with real contours, namely, by adding a real solution to the gaps?

A further demonstration of the logical and phenomenological inadequacy of incompleteness in predicting the way gaps should be completed is illustrated in Figure 14a. Several circumferences with missing arcs are broken even if within the gaps some brightness inductions are perceived allowing one circumference to pass behind the other. A similar effect is perceived in Figure 14b–c. The gaps and the brightness inductions are like "illusory amodal completions." They are not illusory figures in the known sense. They do not show any illusory contour. Moreover, they belong to neither one nor the other circle and do not appear as gaps or incompleteness. This illusory amodal completion is comparable or even stronger than the one where there are no gaps. By increasing the amplitude of the gaps, the strength of the amodal completion and the stratification effect in depth increases accordingly.

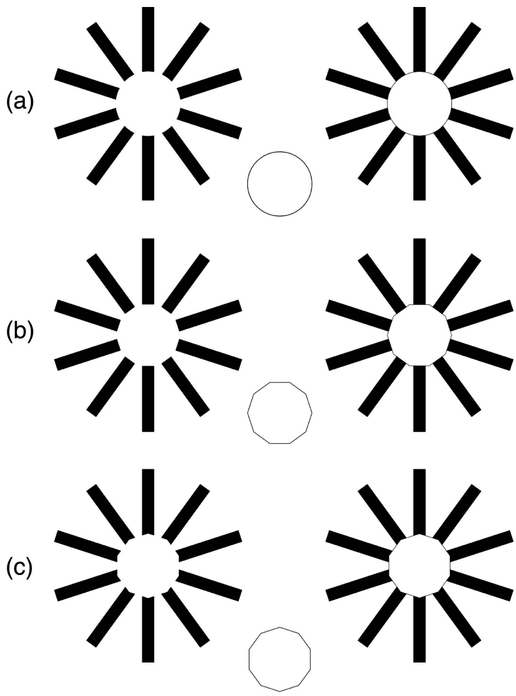
In Figure 14b–c, the illusory amodal completion persists but it assumes a different appearance: the gaps become the white boundaries of the circumferences. It is worth to note that the bright illusory boundaries are perceived only along the crossover points and not everywhere along the circumferences. They are illusory in that they are detached from the black region that they bound. These illusory contours define T-junctions with respect to the real contours from which they are detached. These phenomenal outcomes cannot be predicted from incompleteness assumed as a sufficient condition in order to explain illusory contour.

These outcomes are further corroborated by the more complex conditions illustrated in Figure 14d–f.

Incompleteness Is Not Necessary

The strongest logical condition to be studied is incompleteness as a necessary condition; that is, A is necessary for B if B cannot be true unless A is true. In phenomenal terms: if the inducing elements are complete, then not any illusory figure should be induced or, in other words, a case of complete elements inducing an illusory figure can never happen.

Figure 13
Illusion of Angularity: (a Left) an Illusory Disk; (a Right) the Circle Appears to be Polygonal With Blunt Angles Directed Toward the Inside of the Stripes; (b Left) an Illusory Polygon; (b Right) the Sides of the Polygon Look Convex With Swellings Inside the Stripes and the Vertices Appear Less Pointed, Blunter, and Rounded off; (c Left) an Illusory Polygon; (c Right) the Polygon Appears More Polygonal, Pointed or Sharper With the Polygon Sides Appearing Slightly Concave and the Vertices Seeming to Go Even More Inward Into the Stripes



Within the logical rationale of the previous approaches, incompleteness is completed through depth segregation and amodal completion, therefore (a) whenever an illusory object is seen, necessarily both depth segregation of the illusory figure in relation to the inducing elements and their partial occlusion should also be perceived; (b) there should never be the case of an illusory figure placed on the same depth (coplanar) to its inducing elements.

To refute the necessary condition, three conditions have been created. One is the well-known Sun effect by Kennedy (1976, 1978), made of triangles radially arranged and pointing toward a central open area. Through this interesting pattern, Kennedy demonstrated illusory brightness without the amodal completion of the inducing elements. Although this case can be considered a counter-example, it does not completely disprove the role of incompleteness and completion. In fact, in the Sun effect, the illusory contours are perceived not as sharp as Ehrenstein illusion but fuzzy with weak surface qualities, that is, diaphanous like a bright fog without a defined depth location. The resulting illusory disk seen in Figure 3 is reminiscent of Kennedy's effect. The novelty of Figure 3 is related to the 3D appearance of the inducing elements, phenomenally and logically more powerful and effective than Sun effect.

Purghé (1990) suggested another interesting case to refute the necessary condition. He arranged four black octagons in a way that the central illusory octagon appears as tessellated to the other four placed all around it. This limiting case of amodal completion uses implicit Y-junctions between the illusory figure and the inducing elements. This is also an interesting and clear condition, although the strongest refutation of the necessary condition should be founded on T-junctions between inducing elements and illusory figure.

By replacing Ehrenstein inducing lines by alphabetical letters (either normal or modified) with similar line terminators (Figure 15a–b), Pinna (1996) and Pinna et al. (2004) demonstrated a stronger refutation of the necessary logical condition not only by means of the T-junctions but also by virtue of the perception of alphabet letters requiring a cognitive process defining their completeness. Although, each letter is perceived complete both perceptually and cognitively, their radial arrangement induces a strong illusory circle, thereby against incompleteness as a necessary factor. Moreover, the illusory circle is perceived coplanar and tangent to the letter terminators. Finally, an illusory disk with sharp contours is perceived even if curved terminators (R and B) and line-end terminators are mixed.

The three logical and phenomenal conditions are also perceived by rotating Figure 15b in 3D space as shown in Figure 15c. The strength and sharpness of illusory contours and the coplanarity of the illusory disk appear now stronger than previously. The illusory figure persists also with misalignments and deformations both in the inducing letter and in the illusory contours (Figure 15d–f).

Figure 14

(a–c) Inadequacy of Incompleteness to Predict How Gaps Should Be Completed: (a) a Circumference With Two Missing Arcs, (b) the White Gaps Become "Something" (Neither an Illusory Figure Nor a Gap) That Allows One Circumference to Pass Behind the Other, (c) "Something" That Induce the Incomplete Circumference to Be Amodally Completed Behind the Complete Circle, (d) the Amodal Completion Is Less Strong Than in (b) and (c), (e) the Gaps Become the White Illusory Boundaries of the Circumferences That Appear as Surfaces, (d–f) Further Examples of the Inadequacy of Incompleteness to Predict How Gaps Should Be Completed

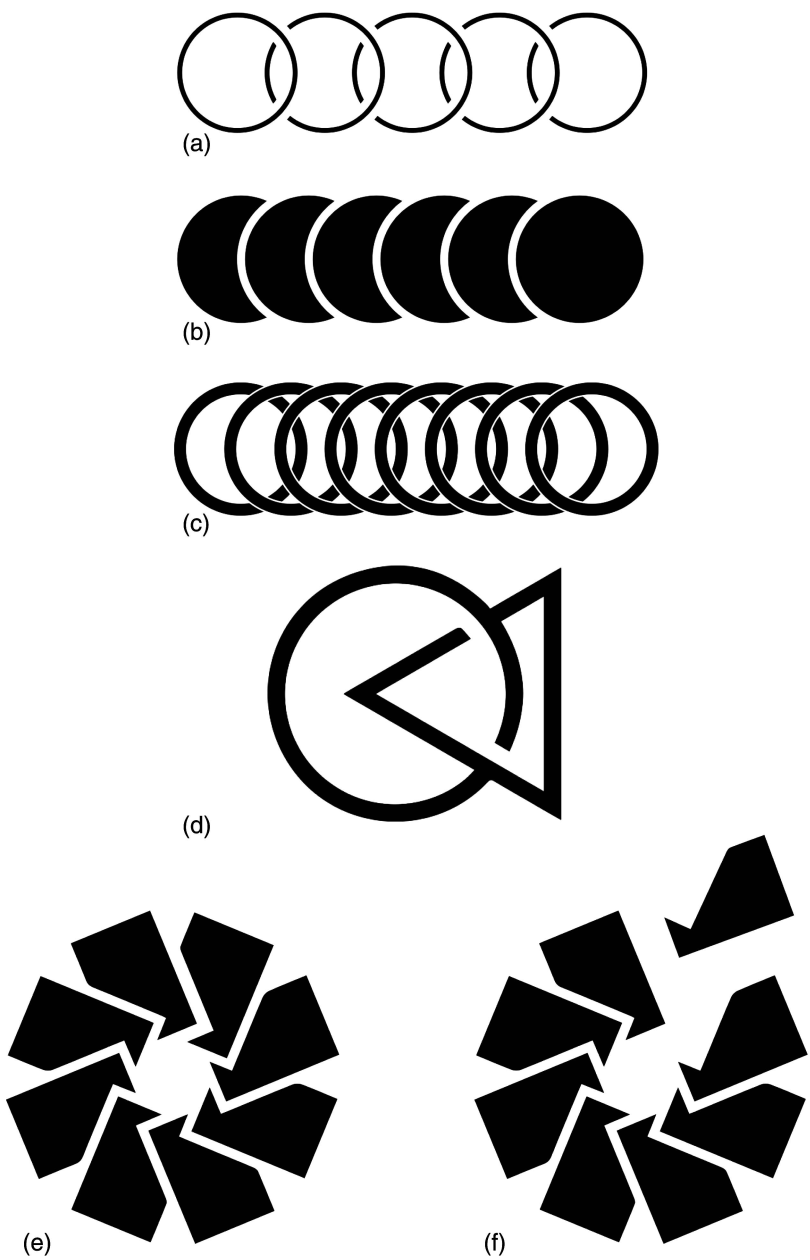
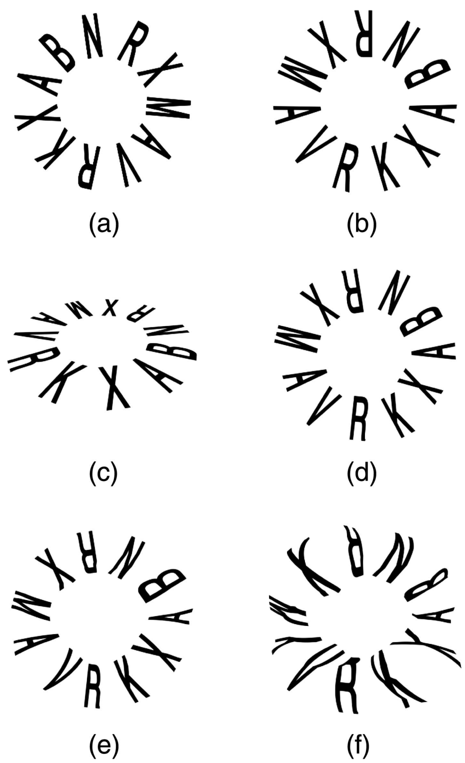


Figure 15

Illusory Figures Without Incomplete Inducing Elements: (a) Each Letter Is Perceived Complete, but Their Radial Arrangement Induces a Strong Illusory Circle, and Furthermore, the Illusory Circle Is Perceived Coplanar and Tangent to the Letter Terminators, and the Illusory Circles Persist Even When Each Letter Is Upside Down (b), Tilted in 3D Space, (c) and When Both the Inducing Letter and the Illusory Contours Are Misaligned (d) and Deformed (e and f)



These results clearly demonstrate that incompleteness is neither a necessary nor a sufficient condition for inducing illusory figures, and suggest a different explanation based and useful to understand the general problem of figural organization.

How can the previous results be interpreted? Can incompleteness be replaced by a more effective and simpler hypothesis addressing its logical and phenomenological issues? Previously, we have hinted several times to a possible role of grouping and figure-ground segregation principles. In the next sections, we discuss figural organization in terms of how percepts are completed in response to boundary contour discontinuities and brightness inhomogeneities.

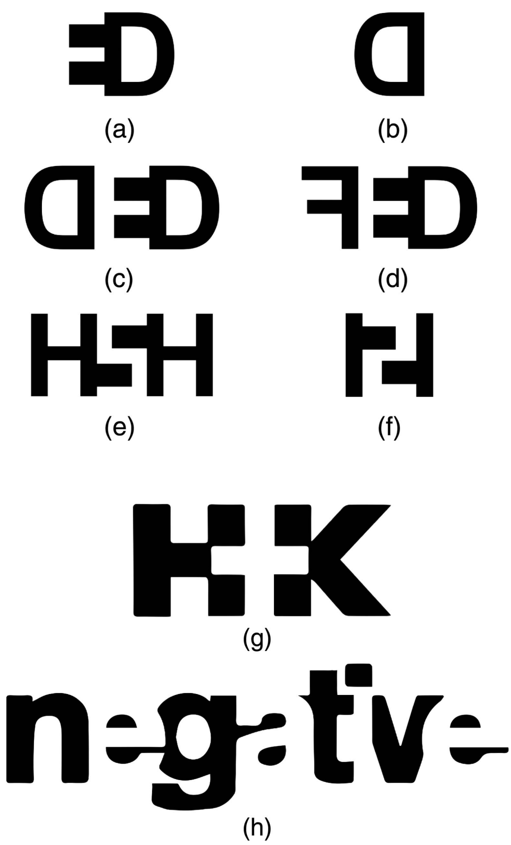
Figure-Ground Organization of Contour Discontinuities

The problem of figure-ground segregation is based on the formation of the following complementary qualities (Koffka, 1935; Köhler, 1920; Metzger, 1963; Rubin, 1915, 1921) (a) a border is unidirectional, and belongs to the figure (border ownership), not to the ground; (b) the figure manifests a bright, compact, and opaque surface color, whereas the ground shows a diaphanous color and appears empty; and (c) a figure appears to lie above the ground, whereas the ground is unlimited and continues underneath the figure.

These points are illustrated in Figure 16. In Figure 16a, an electric plug-like shape is perceived, or more geometrically two horizontal rectangles, connected with the shape of Figure 16b. By putting together Figure 16a–b, as illustrated in Figure 16c, a strong illusory E is perceived and the whole pattern is read as DED. While the two Ds are perceived on a white ground, the E is placed on a black ground. Incompletenesses are totally absent in the element components. More importantly, the specific elements can be defined only a posteriori, when the illusory figure is perceived. This implies again the paradox already discussed, according to which incompleteness can be defined and perceived on the basis of the illusory figure and vice versa. As a matter of fact, convexities, clearly perceived in Figure 16a, are not perceived as such in Figure 16c, since their boundaries belong to the E and not to the plug-like shape, thus they become background.

Similar examples useful for a comparison of the strength of the illusory letters are illustrated in Figure 16d–e, where convexities are complementary to concave discontinuities and gaps. The illusory E is clearly perceived in Figure 16d and the illusory 5 in Figure 16e. While the concave discontinuities (for example the three white arms of the E in Figure 16d) tend to appear as a figure, the convex additions to the D in Figure 16d are perceived as a black background. Since the same border can be perceived both concave or convex on the basis on the direction of the perception, namely, from left to right or from right to left, the border of the two conditions appears belonging to complementary regions. Furthermore, the two black additions to the D in Figure 16d, perceived as a black background, are not perceived as such and, as a consequence,

Figure 16
(a–f) Figure-Ground Organization of Boundary Contour Discontinuities: (a) Electric Plug-Like Shape, (b) Half-Circular Shape, (c) by Putting Together (a) and (b) a Clear Illusory E is Perceived and the Whole Pattern Is Read as DED, (d) an Illusory E Is Perceived, (e) an Illusory 5 Digit Emerges, (f) an Illusory Number 2 on a Black Background or, Alternatively, an Illusory 2 on a Black Background With Two Adjacent I Letters on Both Sides of the 2 Digit, and (g–h) Figure-Ground Segregation and Illusory Figures



they split from the D, inducing two illusory vertical boundary contours that separate the two regions belonging to the D. These illusory contours disappear when the electric plug-like shape is perceived.

These simple examples corroborate the hypothesis that illusory figures can be assumed as part of the more general problem of figure-ground segregation and the way boundaries are grouped and ungrouped depends on figure-ground constraints.

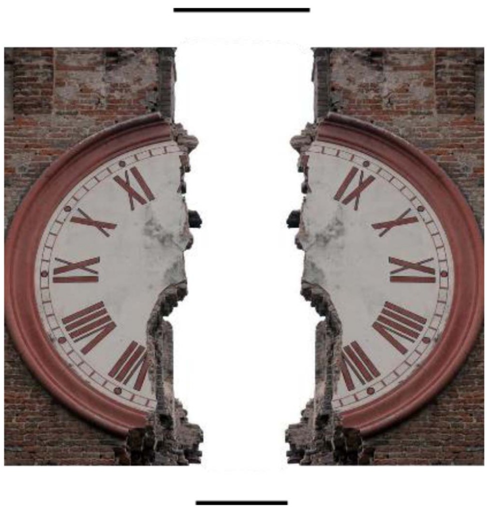
Figure-ground organization plays a clear role even when the effects of past experience are weakened. More particularly, grouping can influence border ownership. Figure 16f can be seen as (a) an illusory number 2, and an illusory reflected S both on a black background or (b) an illusory number 2 on a black background with two adjacent I letters on both its sides, that is, I2I. The inducing elements, could be the two small horizontal rectangles. The percept of the 2 on black background is related to the grouping of the small horizontal rectangles with the two larger vertical ones. The percept of I2I is made easy by ungrouping the small rectangles from the larger ones, therefore eliciting the illusory contour as described in Figure 16d. The ungrouping makes the perception of the illusory 2 easier, enabling the small rectangles to lose and leave their boundaries to the illusory 2 digit and enabling the large rectangles to appear as such or like I letters, according to past experience.

Figure 16 g–h shows more clearly the inner relation between illusory figures and figure-ground segregation. The illusory white symbol and letters further corroborate and reinforce the previous outcomes.

The role of perceptual organization is further supported by Figure 17, where two different phenomenal outcomes are mostly perceived: (a) a broken clock tower with two dashes placed at the top and bottom of the empty space in between the two halves and (b) the empty space of the previous description appears as some sort of illusory vertical stripe seen transparent in between the two broken parts of the clock tower. Both descriptions do not show incompleteness but broken parts, not requiring to be completed. These results can be accounted for by invoking grouping principles that put together and find a common meaning to all the elements involved.

The role of figure-ground segregation in eliciting illusory figures can be partially demonstrated in some variants of Rubin's vase/cup-face profiles (see Figure 18). Figure 18a shows "a black cup." However, after a prolonged viewing two illusory close white face profiles, facing each other, suddenly pop out like illusory figures. When they emerge, the cup "disappears" becoming invisible, simply background, that is, nothing, not a figure. Once perceived, these two possible outcomes can be alternated by switching the visual attention on one or on the other result (Peterson et al., 1991; Peterson & Gibson, 1993, 1994). The relative salience can be easily inverted by reversing the contrast, as illustrated in Figure 18b. Now, the profiles pop out more strongly and spontaneously,

Figure 17
A Broken Clock Tower With Two Dashes Placed at the Top and Bottom of the Empty Space in Between the Two Halves



Note. The empty space can be perceived assome sort of illusory vertical stripe seen as transparent in between the two broken parts. See the online article for the color version of this figure.

while the cup is invisible or barely perceptible. In Figure 18c, the figural salience of the two possible results is closely balanced without any illusory figure. The result is highly reversible, although the human bias to see faces (pareidolia) puts the cup at a disadvantage.

A more striking effect is illustrated in Figure 19. Although this figure and the resulting phenomena are reminiscent to Rubin's vase/cup-face profiles, it is structurally very different and more effective for our purposes. In Figure 19a, an illusory bottle is clearly perceived in between the incomplete frontal faces of two mirrored ancient Roman statues, specularly arranged. It is noteworthy that the bottle can be seen even though the two faces appear incomplete, given their salient depth and 3D cues with shaded borders and cast shadows.

As a control, in Figure 19b, the incompleteness of only one face can be better appreciated without illusory figure.

The resulting illusory bottle can be made more salient by including details within its surface as shown in Figure 19c–d.

The complex role of illusory figures and figure-ground segregation and grouping principles is illustrated in Figure 20. Here, differently from the previous conditions shown in Figures 18 and 19, the main issue is related to the weak role of past experience against other more powerful principles, under these conditions. They are closure and articulation-without-rests.

In Figure 20a, a black silhouette of an incomplete man is depicted with head, one hand, ankles, and feet missing. The presence of the two ellipsoidal dots on the bottom of Figure 20b induces some sort of illusory object similar to a white step partially occluding the lower parts of the man: his ankles and feet. In Figure 20c, the two dots placed near the missing head and hand, induces or highlight illusory cylindrical figures in place of the head and the hand. More precisely, they become the head and the hand.

The alphabet letters are here placed on the left side of each silhouette since they can induce effects similar to the ellipsoid dots.

It is noteworthy that this kind of illusory figure can be made more effective in the conditions

Figure 18
Rubin's Vase/Cup-Face Profiles With Variations and Figure-Ground Segregation Inducing Illusory Figures

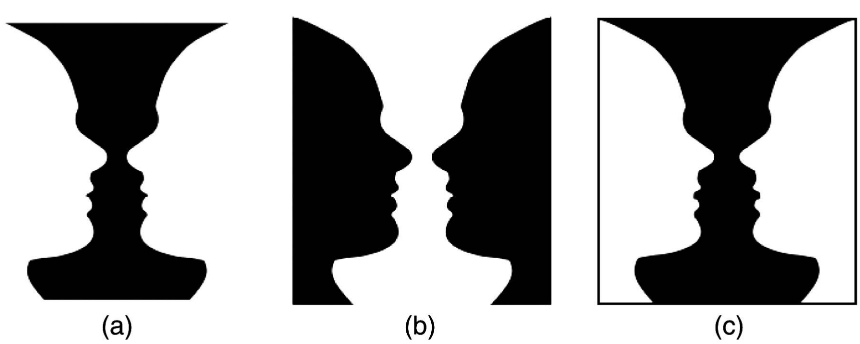
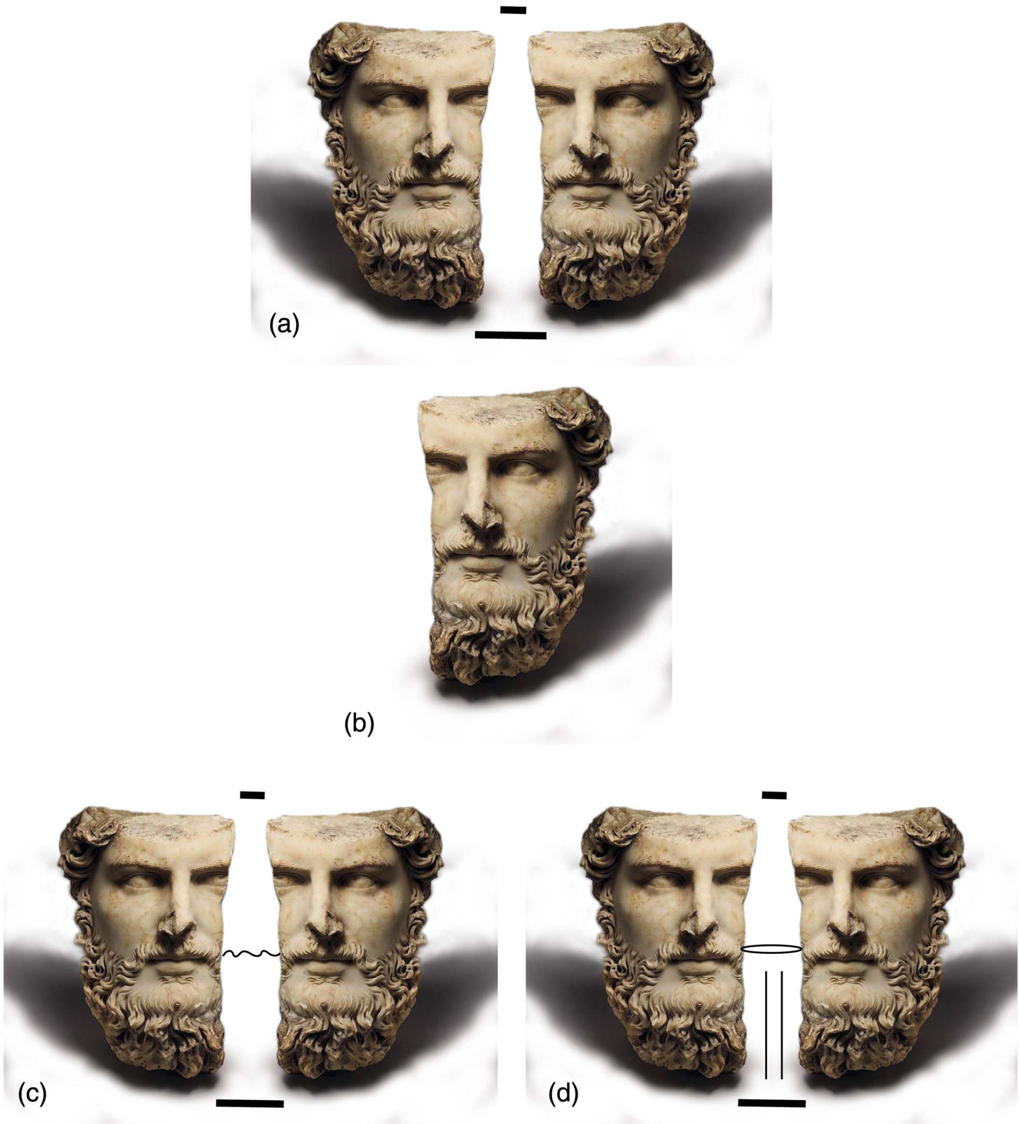


Figure 19

(a) An Illusory Bottle Perceived in Between the Incomplete Frontal Faces of Two Identical Ancient Roman Statues, Specularly Arranged, (b) a Control for (a), and (c, d) the Illusory Bottle Is More Salient Than the One Shown in (a)



Note. See the online article for the color version of this figure.

illustrated in Figure 20d–g, where the head, hand, ankles, and feet can be elongated or reduced according to the limits imparted by the dots.

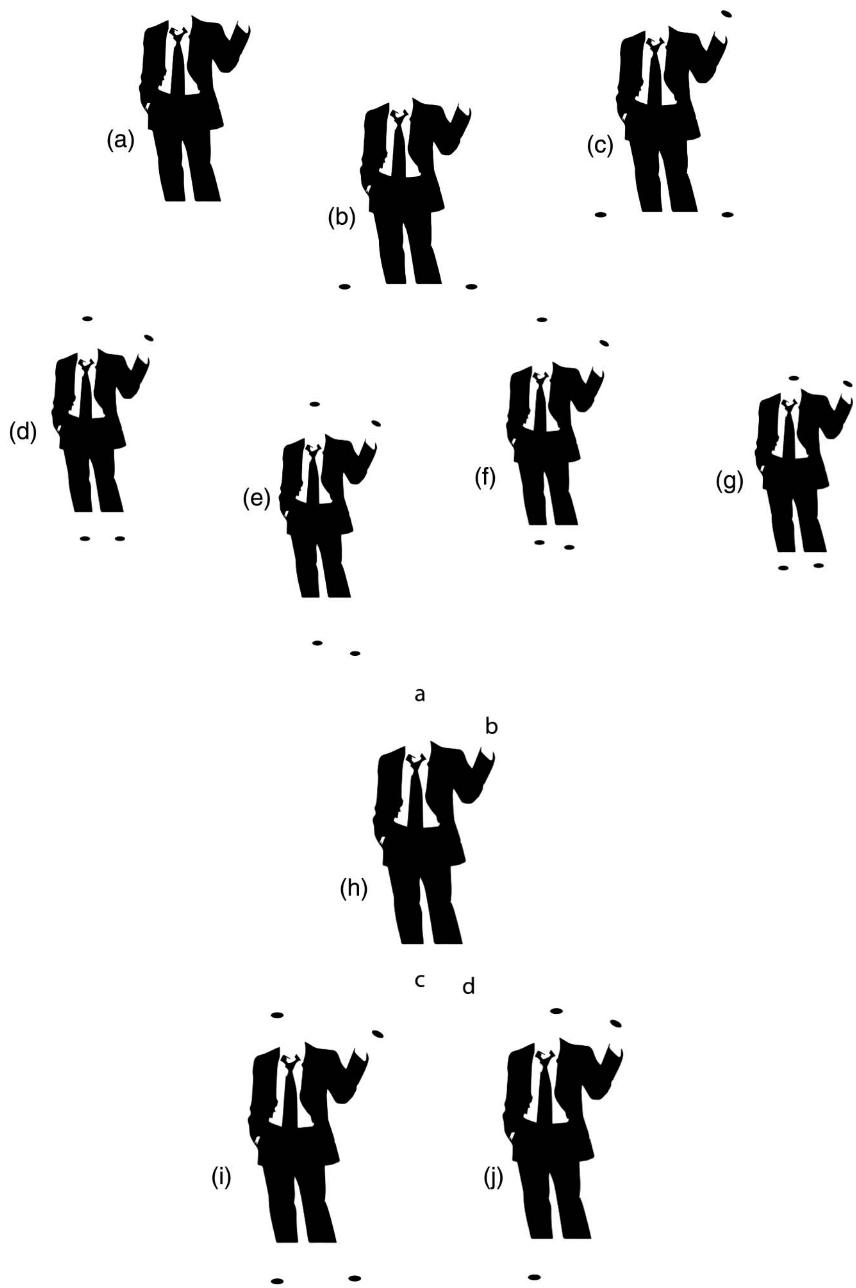
Here, the letters are removed, however when they replace the dots, the same illusory phenomena are restored although less saliently (Figure 20 h).

In Figure 20i–j, the man is perceived to change his posture.

Taken together, all these phenomenal outcomes demonstrate that also incompleteness can be explained in terms of grouping processes and figure-ground organization. Such a hypothesis avoids the paradoxes raised by incompleteness if assumed as an explanatory principle. As a matter of fact, grouping principles: (a) are not restricted to specific figural conditions like

Figure 20

(a–c) (a) A Black Silhouette of an Incomplete Man With Head, One Hand, Ankles and Feet Missing; (b) An Illusory Object Similar to a White Step Partially Occluding the Lower Parts of the Man: Ankles and Feet; (c) Illusory Cylindrical Figures in Place of the Head and the Hand; (d–g) the Illusory Head, Hand, Ankles and Feet Can Be Elongated or Reduced According to the Limits Imparted by the Dots; (h) the Letters Induce the Same Illusory Phenomena Although Less Saliently Than the Ones of Figure 20a–g; and (i–j) the Illusory Man Is Perceived to Change His Posture



incompleteness but cover a multiplicity of boundary contingencies, like additions, line terminators and parallel contours; (b) incorporate both local and global boundary conditions; (c) do not necessarily require amodal completion of inducing elements; (d) can predict different or complementary figure-ground organizations, as previously described; (e) predict the shape of illusory figures on the basis of the grouping of local discontinuities and their connections; (f) the shape of illusory contours is not expected to be necessarily equal to the shape of real contours and to the shape of incompleteness.

The figural properties of the illusory figures are accompanied by brightness enhancements. Both boundary grouping and surface filling-in processes work in parallel and are represented by parallel cortical interblob and blob streams from cortical areas V1 through V4, to synergistically create the strong figural properties of real and illusory figures. Grossberg (1994, 1997), Grossberg & Mingolla (1985a, 1985b), Kelly and Grossberg (2000), and of Raizada and Grossberg (2003) proposed and simulated cortical mechanisms of perceptual grouping and figure-ground perception whereby these properties might be actualized.

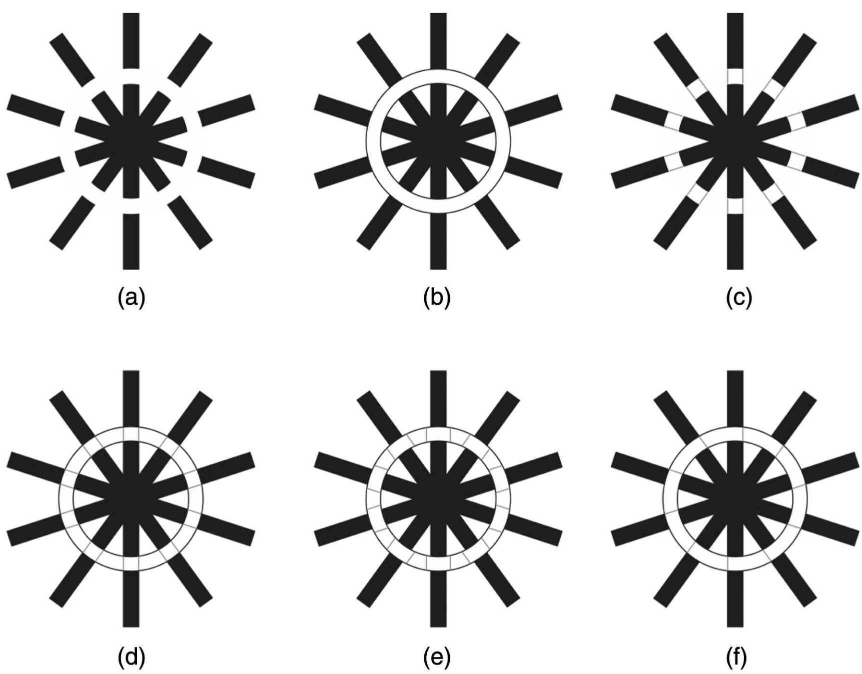
Figural Organization of Brightness Induced Inhomogeneities

In Figure 21a, the radial black elements appear partially occluded by an illusory annulus. By replacing the illusory contours with real ones (see Figure 21b), the annulus loses the brightness enhancement. However, some weak brightness enhancement persists within the interspace between the terminators of the radial components. A further effect is the illusion of angularity (Pinna, 1991), described in Figure 13.

In Figure 21c, the brightness enhancement is stronger than in Figure 21b, due to the pairs of lines that continue the boundaries of the stripes

Figure 21

Figural Organization of Brightness Inhomogeneities: (a) the Radial Stripes Appear Partially Occluded by an Illusory Bright Annulus; (b) the Annulus Loses Its Large Brightness Enhancement, Although Weak Brightness Enhancement Persists Within the Interspace Between the Stripe Terminators; (c) the Brightness Enhancement Is Stronger Than in (b); (d) the Induced Brightness Next to the Stripe Terminators Pops Out Relative to the Dark Connecting Regions Within the Annulus; and (e) and (f) the Brightness and Darkness Spreading Effects Are Diluted, Though Not Homogeneously



within the interspaces and surround the brightness induction. In Figure 21d, bordering the annulus of Figure 21c with real contours, the induced brightness, adjacent to the stripe terminators, pops out and appear brighter than the dark inner regions within the remaining part of the annulus. Therefore, brightness and darkness effects are contained in separated sectors of the annulus.

Behaving as filling-in generators and filling-in barriers (Grossberg, 1994, 1997), the separation segments stop and contain both brightness and darkness filling-in within the annulus. By expanding the brightness region, as shown in Figure 21e–f, the brightness and darkness effects are diluted, although not homogeneously (see also the anomalous brightness differentiation (Kanizsa & Minguzzi, 1986) and the impossible staircase brightness illusions (Escher, 1961; Penrose & Penrose, 1958).

The main conclusions of this section are the following: (a) brightness inductions peculiar to illusory figures are induced whether or not figural incompletenesses are present; (b) brightness enhancement is induced next to each element terminator; (c) brightness inhomogeneities are induced independently from illusory or real contours; (d) brightness inhomogeneities spread and fill-in long range distances; (e) brightness spreading can be contained by both illusory and real boundaries functioning as barriers; (f) darkness enhancement may be perceived, especially when a real boundary separates the respective regions; (g) bright and dark inhomogeneities mix while they spread if they are not separated by barriers.

The amount of brightness and darkness induction changes by replacing illusory with real contours but in a manner not depending upon incompletenesses. Going from Figure 21a to Figure 21b, the decrease of brightness may depend on darkness assimilation due to the parallel black lines among the radial stripes as supported by Figure 21d. Now, the assimilation effect is enhanced due to the pairs of perpendicular parallel lines enclosing the darkness regions and separating them from the brightness areas. The hypothesis of darkness induction due to assimilation processes might be only a piece of the explanation of the outcome. As a matter of fact, it appears phenomenally similar to the dark spots seen peripherally at the crossroads of the Hermann grid and in the inner circular arrangement of triangles of Kennedy's Sun Effect.

These figures, among others (Brigner & Gallagher, 1974; Day & Jory, 1978; Frisby & Clatworthy, 1975; Grossberg, 1983, 1984, 1987a, 1987b; Jory & Day, 1979; Grossberg & Mingolla, 1985a, 1985b, 1987), demonstrate that surface processing is a distinct dynamic occurring parallel to boundary formation, and not necessarily derived from incompleteness.

Figures 22–28 illustrate this conclusion in a very effective way. In the classical Ehrenstein illusion (Figure 22-top), brightness enhancement fills the central gaps between the radial lines. Ehrenstein (1941) demonstrated that, by superimposing black rings onto the illusory disks, the brightness effect largely disappears (Figure 22-bottom). This outcome is in agreement with incompleteness approaches. Nevertheless, when colored annuli (e.g., purple or cyan, as in Figures 23–28) replace the black rings, the brightness cancellation of Figure 22-bottom does not occur. On the contrary, another effect emerges, much brighter than in the Ehrenstein illusion. The white disks also show a dense appearance comparable to a paste of bright and quasi-luminous white color onto the paper surface (cfr. Pinna et al., 2003). Further examples of brightness induction with completed gaps are illustrated in Figures 22–25. Figure 22 is a control.

Figure 22

(Top) Brightness Enhancement Fills the Central Gaps Between the Radial Lines (Ehrenstein Illusion) and (Bottom) When Black Rings Are Superimposed Onto the Illusory Disks, the Brightness Enhancement Largely Disappears

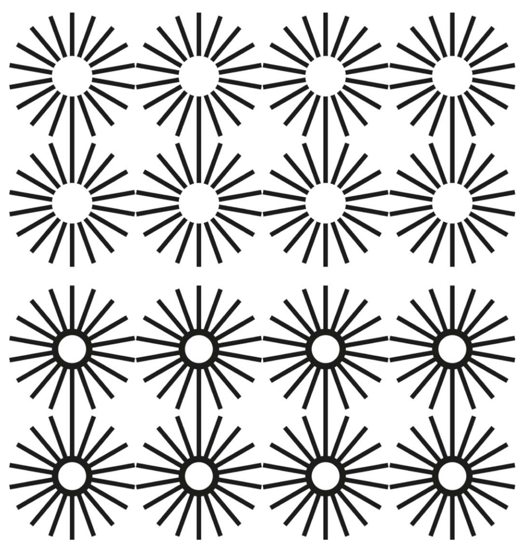
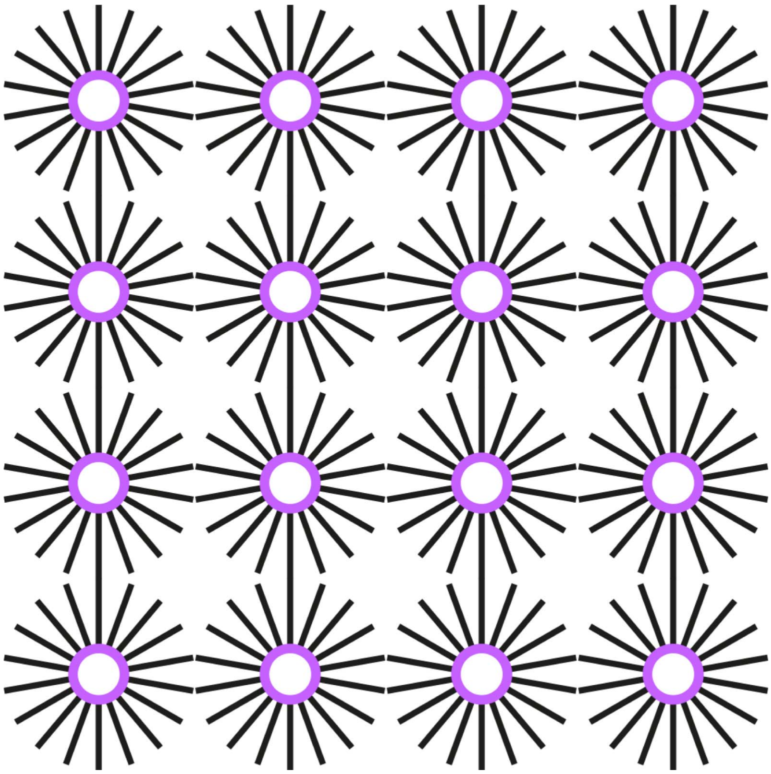


Figure 23
Brightness and Luminous Illusion Induced by Purple Annuli



Note. See the online article for the color version of this figure.

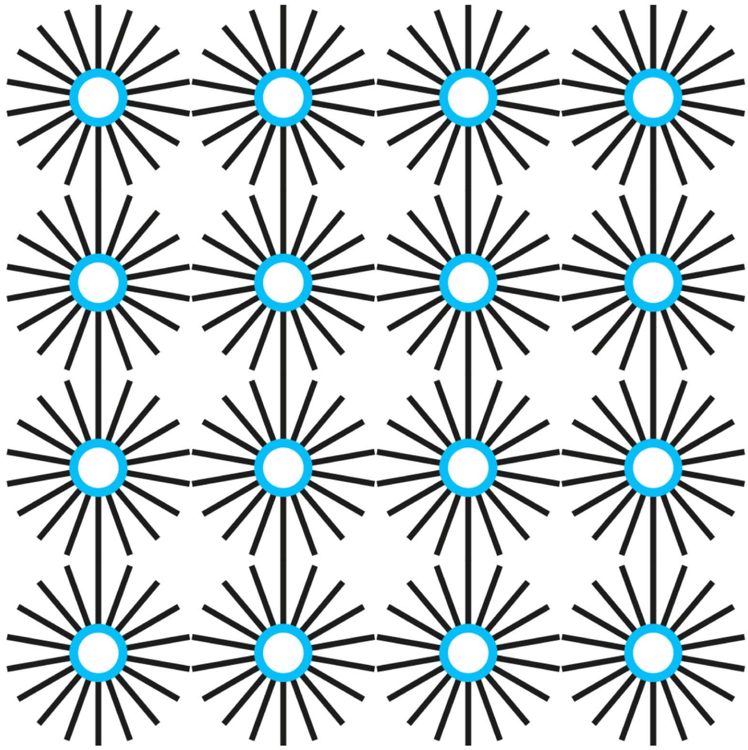
The cases illustrated in this section demonstrate that boundary and surface formation processes are parallel, distinct, but mutually interactive processes that together give rise to figure-ground segregation properties. Indeed, brightness inhomogeneities are especially induced next to line terminators, as well as concave and convex boundary discontinuities.

Discussion and Conclusion

Gestalt and cognitive approaches considered incompleteness as one of the basic factor for illusory figure formation. The role of incompleteness has been explored in terms of its inner logic and through an accurate phenomenal analysis of counterexamples and limiting or critical conditions. They demonstrated a bunch of logical issues and paradoxes, and, more importantly, that incompleteness is neither sufficient nor necessary to induce illusory figures. These issues are strongly reduced a possibly solved when the incompleteness is replaced by more simple concepts concerning interacting boundaries, grouping, and surface filling-in processes during figure-ground segregation. These dynamics give rise to the three basic figure-ground properties as deeply studied by Gestalt psychologists: the unidirectional belongingness of boundaries to the illusory figure, not to the background; the enhanced brightness of surface color under certain conditions; and the illusory figure lying above a background that continues underneath the illusory figure.

Compared to the logical and phenomenological problems coming from the incompleteness hypothesis, advantages of this hypothesis are: (a) incompleteness becomes just one property to be explained, analogously to illusory figures, again in terms of boundary and surface dynamics; (b) these dynamics are not restricted to incompleteness and are not subjected to the paradoxes of incompleteness; (c) they can handle other kinds of discontinuities than incompleteness; (d) they

Figure 24
Brightness and Luminous Illusion Induced by Cyan Annuli



Note. See the online article for the color version of this figure.

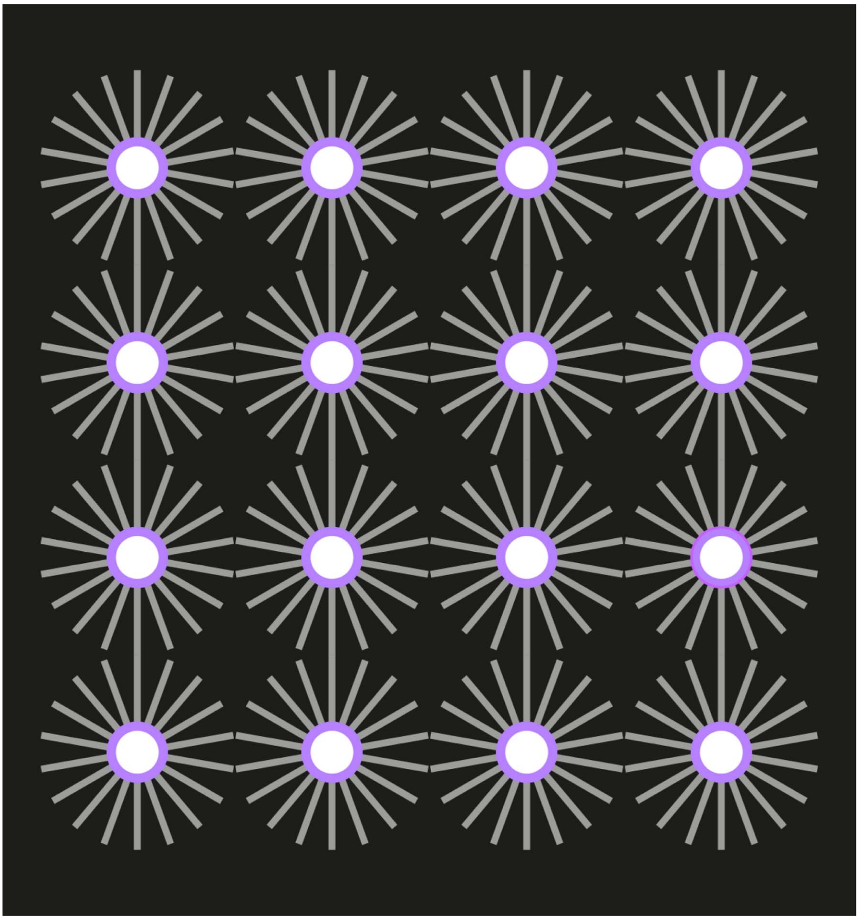
might explain both local and global levels of object representation, including both bottom-up and top-down levels.

The suggested hypothesis is consistent with neurophysiological experiments. von der Heydt et al. (1984) and Peterhans and von der Heydt (1987) demonstrated that neurons in area V2 of the monkey cortex respond to both real and illusory contours. Sasaki and Watanabe (2004) reported distinct fMRI signatures in the human visual cortex for illusory contours and for color spreading processes, including color spreading in V1. Zhou et al. (2000) demonstrated a substantial fraction of cells that are sensitive to border-ownership in area V2. Baylis and Driver (2001) found that neurons in monkey IT (inferotemporal) cortex, which are supposed to be involved in object recognition, respond differentially to figure or ground, and as a consequence to border ownership. Kleinschmidt et al. (1998) in fMRI studies on figure-ground reversals

Figure 25
A Control for the Previous and Following Figures

Note. See the online article for the color version of this figure.

Figure 26
Brightness and Luminous Illusion



Note. See the online article for the color version of this figure.

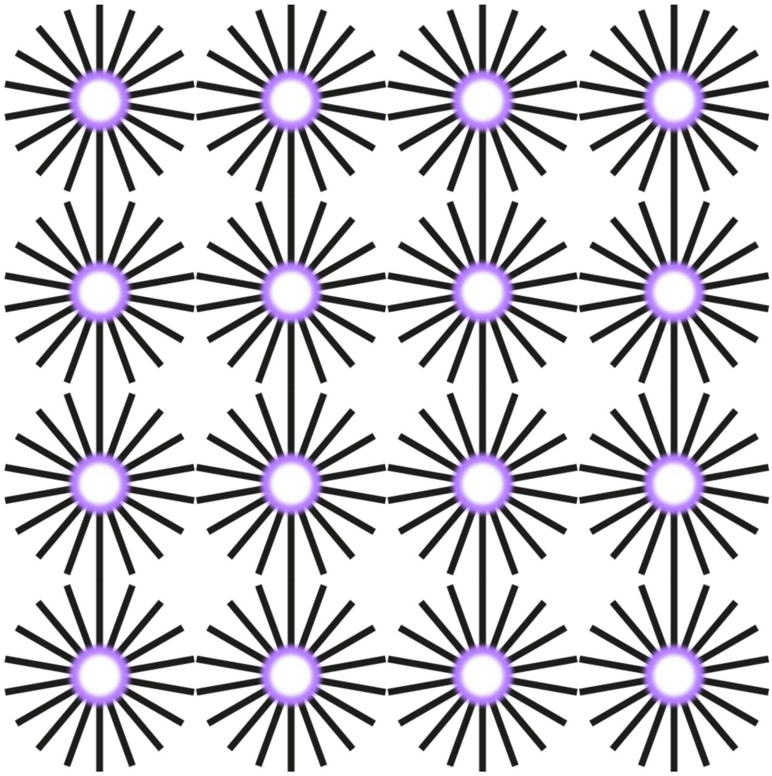
found activation over a number of areas in occipital, temporal, parietal, and even frontal cortex. fMRI studies of Kanizsa squares by Hirsch et al. (1995) revealed that there was activation of the occipital cortex lateral to V1 where signals related to segmentation were present. Mendola et al. (1999) found that signals related to illusory contours were observed in cortical area V3 and also in LO, the lateral occipital area (Malach et al., 1995).

The FACADE neural model of boundary and surface formation during figure-ground segregation (Grossberg, 1994, 1997; Grossberg & Yazdanbakhsh, 2005; Kelly & Grossberg, 2000; Raizada & Grossberg, 2003) provides a conceptual foundation useful to replace incompleteness with perceptual and neural organizational principles and mechanisms.

Comments on Bayesian Framework

Our results weaken Bayesian probability theory as a candidate to explain illusory figures. Based on Helmholtz's likelihood, the Bayesian statistical decision theory formalizes mathematically the idea of perception as inference. This theory is considered as an optimal method for making decisions under conditions of uncertainty similar to those inducing illusory figures (Bülthoff & Yuille, 1991; Feldman, 2009; Knill & Richards, 1996; Landy et al., 1995; Liu et al., 1995;

Figure 27
Fuzzy Brightness and Luminous Effect



Note. See the online article for the color version of this figure.

Mamassian & Landy, 1998; Nakayama & Shimojo, 1990; Weiss & Adelson, 1998).

As a matter of fact, the Bayesian framework provides an elegant way of dealing with all these uncertainties. This is why these techniques are more and more popular not only in building artificial systems in the face of uncertainty but also in developing a theory on how the brain works and deals with uncertainties.

Furthermore, in machine learning, the main interest is to determine the best hypothesis from some space of hypothesis, given the observed training data. In Bayesian analysis, we can use previous information, either belief or experimental evidence, in a data model to acquire more balanced results for a particular problem. As a matter of fact, by using the knowledge of the entire posterior distribution of model parameters, Bayesian inference is far more comprehensive and flexible than the traditional inference.

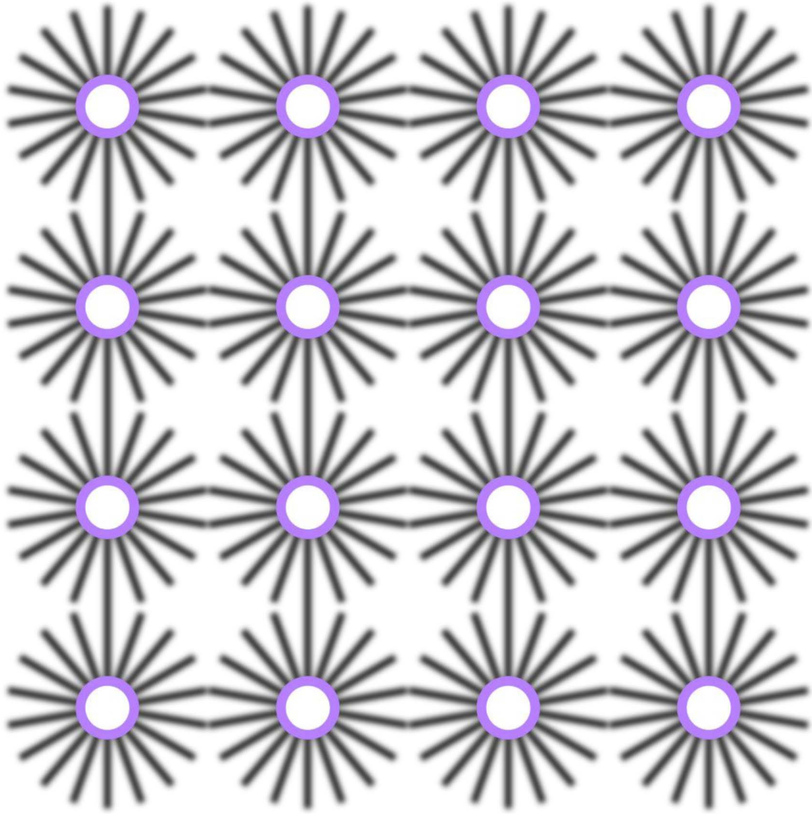
Bayesian inference is also exact, that is, estimation and prediction are based on the posterior distribution, known analytically or estimated numerically with arbitrary precision, and provides a straightforward and more intuitive interpretation of the outcome.

As such, Bayesian approach has clear advantages over other approaches in perception by incorporating anatomical, physiological, and also phenomenological constraints, which become good preliminaries to develop models more and more useful.

Despite these conceptual and methodological advantages, there are also disadvantages highlighted by our phenomenal results and logical argumentations.

Along with the objectivity that comes from the data, the Bayesian inference is based on potentially subjective prior distribution. Different prior distributions can be specified but it remain subjective. As a matter of fact, building a reliable

Figure 28
Brightness and Luminous Illusion



Note. See the online article for the color version of this figure.

Bayesian model requires extensive experience: setting up a model and performing analysis is highly demanding and source of significant subjectivity. In other words, Bayesian approach is considered highly controversial given the presumed subjectivity in specifying prior information and the computational load in implementing Bayesian methods. It does not contain any correct way and information about how to select a prior. Moreover, Bayesian inferences require high skills and caution to translate subjective prior beliefs into a mathematically formulated prior. Misleading outcomes and predictions can be produced otherwise. It can also produce posterior distributions that are heavily influenced by the priors. Within the context of illusory figures, the problem with the priors is exponentially amplified by assuming the notion of incompleteness as the basic factor of this kind of illusion.

Another important issue is the relation between falsificationism and Bayesian statistics. Although the application of Bayes' theorem recovers the essence of Popper's falsificationism, all non-falsified hypotheses, which in falsificationism are waiting to be considered, in the Bayesian approach have different degrees of beliefs depending on all available information. Moreover, it is necessary to distinguish between what is impossible and what is very improbable. This issue is effective within the logic and phenomenology discussed in the previous section.

In conclusion, we have demonstrated that the empirical evidence for Bayesian theories to explain illusory figures is weak. This weakness mainly depends on the notion of incompleteness implying the many arbitrary ways that priors, likelihoods, and utility functions can be altered in order to account for the data, making the models unfalsifiable. Similar issues and weaknesses can be reported for the simplicity principle as used by Kanizsa in accounting for the illusory figures.

References

Atick, J. J., & Redlich, A. N. (1990). Towards a theory of early visual processing. *Neural Computation*, *2*(3), 308–320. https://doi.org/10.1162/neco.1990.2.3.308

Attneave, F. (1954). Some informational aspects of visual perception. *Psychological Review*, *61*(3), 183–193. https://doi.org/10.1037/h0054663

Attneave, F., & Frost, R. (1969). The determination of perceived tridimensional orientation by minimum criteria. *Perception & Psychophysics*, *6*(6), 391–396. https://doi.org/10.3758/BF03212797

Barlow, H. B., Kaushal, T. P., & Mitchison, G. J. (1989). Finding minimum entropy codes. *Neural Computation*, *1*(3), 412–423. https://doi.org/10.1162/neco.1989.1.3.412

Baylis, G. C., & Driver, J. (2001). Shape-coding in IT cells generalizes over contrast and mirror reversal, but not figure-ground reversal. *Nature Neuroscience*, *4*(9), 937–942. https://doi.org/10.1038/nn0901-937

Blakemore, C. (1990). *Vision: Coding and efficiency*. Cambridge University Press.

Bozzi, P. (1989). *Fenomenologia sperimentale*. IL Mulino.

Bradley, D. R., & Dumais, S. T. (1975). Ambiguous cognitive contours. *Nature*, *257*(5527), 582–584. https://doi.org/10.1038/257582a0

Bradley, D. R., & Lee, K. (1982). Animated subjective contours. *Perception & Psychophysics*, *32*(4), 393–395. https://doi.org/10.3758/BF03206245

Bradley, D. R., & Petry, H. M. (1977). Organizational determinants of subjective contour: The subjective Necker cube. *The American Journal of Psychology*, *90*(2), 253–262. https://doi.org/10.2307/1422047

Brigner, W. L., & Gallagher, M. B. (1974). Subjective contour: Apparent depth or simultaneous brightness contrast? *Perceptual and Motor Skills*, *38*(Suppl. 3), 1047–1053. https://doi.org/10.2466/pms.1974.38.3c.1047

Bülthoff, H., & Yuille, A. (1991). Bayesian models for seeing shapes and depth. *Comments on Theoretical Biology*, *2*(4), 283–314. https://hdl.handle.net/11858/00-001M-0000-0013-EE47-A

Chaitin, G. J. (1969). On the length of programs for computing finite binary sequences. *Journal of the Association for Computing Machinery*, *16*(1), 145–159. https://doi.org/10.1145/321495.321506

Chater, N. (1996). Reconciling simplicity and likelihood principles in perceptual organization. *Psychological Review*, *103*(3), 566–581. https://doi.org/10.1037/0033-295X.103.3.566

Coren, S. (1972). Subjective contours and apparent depth. *Psychological Review*, *79*(4), 359–367. https://doi.org/10.1037/h0032940

Day, R. H., & Jory, M. K. (1978). Subjective contours, visual acuity, and line contrast. In J. C. Armington, J. E. Krauskopf, & B. R. Wooten (Eds.), *Visual psychophysics: Its physiological basis* (pp. 331–340). Academic Press. https://doi.org/10.1016/B978-0-12-062260-3.50032-0

Ehrenstein, W. (1941). Über Abwandlungen der L. Hermannschen Helligkeitserscheinung. *Zeitschrift für Psychologie*, *150*, 83–91.

Ehrenstein, W. H., Spillmann, L., & Sarris, V. (2002). Gestalt issues in modern neuroscience. *Axiomathes*, *13*(3/4), 433–458. https://doi.org/10.1023/B:AXIO.0000007203.44686.aa

Epstein, W., & Hatfield, G. (1994). Gestalt psychology and the philosophy of mind. *Philosophical Psychology*, *7*(2), 163–181. https://doi.org/10.1080/09515089408573118

Escher, M. C. (1961). *The graphic work of M.C. Escher*. Oldbourne Press.

Farné, M. (1968). Alcune osservazioni con linee virtuali e margini quasi percettivi. *Bollettino della Società Italiana di Biologia Sperimentale*, *44*, 1613–1616. https://www.sibsperimentale.it

Feldman, J. (1997). Regularity-based perceptual grouping. *Computational Intelligence*, *13*(4), 582–623. https://doi.org/10.1111/0824-7935.00052

Feldman, J. (2003). Perceptual grouping by selection of a logically minimal model. *International Journal of Computer Vision*, *55*, 5–25. https://doi.org/10.1023/A:1024454423670

Feldman, J. (2009). Bayes and the simplicity principle in perception. *Psychological Review*, *116*(4), 875–887. https://doi.org/10.1037/a0017144

Frisby, J. P., & Clatworthy, J. L. (1975). Illusory contours: Curious cases of simultaneous brightness contrast? *Perception*, *4*(3), 349–357. https://doi.org/10.1068/p040349

Gregory, R. L. (1972). Cognitive contours. *Nature*, *238*(5358), 51–52. https://doi.org/10.1038/238051a0

Gregory, R. L. (1980). Perceptions as hypotheses. *Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences*, *290*(1038), 181–197. https://doi.org/10.1098/rstb.1980.0090

Gregory, R. L. (1987). Illusory contours and occluding surfaces. In S. Petry & G. E. Meyer (Eds.), *The perception of illusory contours* (pp. 81–89). Springer. https://doi.org/10.1007/978-1-4612-4760-9_9

Grossberg, S. (1983). The quantized geometry of visual space: The coherent computation of depth, form, and lightness. *Behavioral and Brain Sciences*, *6*(4), 625–692. https://doi.org/10.1017/S0140525X00017763

Grossberg, S. (1984). Outline of a theory of brightness, color, and form perception. In E. Degreef & J. Van Buggenhaut (Eds.), *Trends in mathematical* *psychology* (pp. 59–86). North-Holland. https://doi.org/10.1016/S0166-4115(08)62080-4

Grossberg, S. (1987a). Cortical dynamics of three-dimensional form, color, and brightness perception: I. Monocular theory. *Perception & Psychophysics*, *41*(2), 87–116. https://doi.org/10.3758/BF03204874

Grossberg, S. (1987b). Cortical dynamics of three-dimensional form, color, and brightness perception: II. Binocular theory. *Perception & Psychophysics*, *41*(2), 117–158. https://doi.org/10.3758/BF03204875

Grossberg, S. (1994). 3-D vision and figure-ground separation by visual cortex. *Perception & Psychophysics*, *55*(1), 48–121. https://doi.org/10.3758/BF03206880

Grossberg, S. (1997). Cortical dynamics of three-dimensional figure-ground perception of two-dimensional pictures. *Psychological Review*, *104*(3), 618–658. https://doi.org/10.1037/0033-295X.104.3.618

Grossberg, S., & Mingolla, E. (1985a). Neural dynamics of form perception: Boundary completion, illusory figures, and neon color spreading. *Psychological Review*, *92*(2), 173–211. https://doi.org/10.1037/0033-295X.92.2.173

Grossberg, S., & Mingolla, E. (1985b). Neural dynamics of perceptual grouping: Textures, boundaries, and emergent segmentations. *Perception & Psychophysics*, *38*(2), 141–171. https://doi.org/10.3758/BF03198851

Grossberg, S., & Mingolla, E. (1987). Neural dynamics of surface perception: Boundary webs, illuminants, and shape-from-shading. *Computer Vision Graphics and Image Processing*, *37*(1), 116–165. https://doi.org/10.1016/S0734-189X(87)80015-4

Grossberg, S., & Yazdanbakhsh, A. (2005). Laminar cortical dynamics of 3D surface perception: Stratification, transparency, and neon color spreading. *Vision Research*, *45*(13), 1725–1743. https://doi.org/10.1016/j.visres.2005.01.006

Grünwald, P. (2000). Model selection based on minimum description length. *Journal of Mathematical Psychology*, *44*(1), 133–152. https://doi.org/10.1006/jmps.1999.1280

Grünwald, P. D., & Grünwald, A. (2007). *The minimum description length principle*. MIT press. https://doi.org/10.7551/mitpress/4643.001.0001

Hatfield, G., & Epstein, W. (1985). The status of the minimum principle in the theoretical analysis of visual perception. *Psychological Bulletin*, *97*(2), 155–186. https://doi.org/10.1037/0033-2909.97.2.155

Hirsch, J., DeLaPaz, R. L., Relkin, N. R., Victor, J., Kim, K., Li, T., ... Shapley, R. (1995). Illusory contours activate specific regions in human visual cortex: Evidence from functional magnetic resonance imaging. *Proceedings of the National Academy of Sciences of the United States of America*, *92*(14), 6469–6473. https://doi.org/10.1073/pnas.92.14.6469

Hochberg, J., & Mcalister, E. (1953). A quantitative approach to figural “goodness.” *Journal of Experimental Psychology*, *46*(5), 361–364. https://doi.org/10.1037/h0055809

Hoffman, D. D. (1998). *Visual intelligence: How we create what we see*. Norton.

Jefferys, W. H., & Berger, J. (1992). Ockham’s razor and Bayesian analysis. *American Scientist*, *80*(1), 64–72. https://www.jstor.org/stable/29774559

Jeffreys, H. (1961). *Theory of probability* (3rd ed.). Clarendon Press.

Jory, M. K., & Day, R. H. (1979). The relationship between brightness contrast and illusory contours. *Perception*, *8*(1), 3–9. https://doi.org/10.1068/p080003

Kanizsa, G. (1955). Margini quasi percettivi in campi con stimolazione omogenea. *Rivista di Psicologia*, *49*(7), 7–30. https://link.springer.com/chapter/10.1007%2F978-1-4612-4760-9_4

Kanizsa, G. (1979). *Organization in vision*. Praeger.

Kanizsa, G. (1980). *Grammatica del vedere*. IL Mulino.

Kanizsa, G. (1991). *Vedere e pensare*. IL Mulino.

Kanizsa, G., & Minguzzi, G. F. (1986). An anomalous brightness differentiation. *Perception*, *15*(2), 223–226. https://doi.org/10.1068/p150223

Kelly, F., & Grossberg, S. (2000). Neural dynamics of 3-D surface perception: Figure-ground separation and lightness perception. *Perception & Psychophysics*, *62*(8), 1596–1618. https://doi.org/10.3758/BF03212158

Kennedy, J. M. (1976). Sun figure: An illusory diffuse contour resulting from an arrangement of dots. *Perception*, *5*(4), 479–481. https://doi.org/10.1068/p050479

Kennedy, J. M. (1978). Illusory contours and the ends of lines. *Perception*, *7*(5), 605–607. https://doi.org/10.1068/p070605

Kleinschmidt, A., Büchel, C., Zeki, S., & Frackowiak, R. S. (1998). Human brain activity during spontaneously reversing perception of ambiguous figures. *Proceedings of the Royal Society of London, Series B: Biological Sciences*, *265*(1413), 2427–2433. https://doi.org/10.1098/rspb.1998.0594

Knill, D. C., & Richards, W. (Eds.). (1996). *Perception as Bayesian inference*. Cambridge University Press. https://doi.org/10.1017/CBO9780511984037

Koffka, K. (1935). *Principles of Gestalt psychology*. Brace and Company.

Köhler, W. (1920). *Die physischen gestalten in ruhe und im stationären Zustand. Eine naturphilosophische Untersuchung*. Vieweg & Shon.

Köhler, W. (1947). *Gestalt psychology: An introduction to new concepts in modern psychology*. Liveright.

Kolmogorov, A. N. (1965). Three approaches to the quantitative definition of information. *Problems of Information Transmission*, *1*(1), 1–7. https://alexa nder.shen.free.fr/library/Kolmogorov65_Three-Approaches-to-Information.pdf

Landy, M. S., Maloney, L. T., Johnston, E. B., & Young, M. (1995). Measurement and modeling of depth cue combination: In defense of weak fusion. *Vision Research*, *35*(3), 389–412. https://doi.org/10.1016/0042-6989(94)00176-M

Leeuwenberg, E., & Boselie, F. (1988). Against the likelihood principle in visual form perception. *Psychological Review*, *95*(4), 485–491. https://doi.org/10.1037/0033-295X.95.4.485

Leeuwenberg, E. L. (1969). Quantitative specification of information in sequential patterns. *Psychological Review*, *76*(2), 216–220. https://doi.org/10.1037/h0027285

Leeuwenberg, E. L. (1971). A perceptual coding language for visual and auditory patterns. *The American Journal of Psychology*, *84*(3), 307–349. https://doi.org/10.2307/1420464

Li, M., & Vitànyi, P. (1997). *An introduction to Kolmogorov complexity and its applications* (2nd ed.). Springer-Verlag. https://doi.org/10.1007/978-1-4757-2606-0

Liu, Z., Knill, D. C., & Kersten, D. (1995). Object classification for human and ideal observers. *Vision Research*, *35*(4), 549–568. https://doi.org/10.1016/0042-6989(94)00150-K

Mach, E. (1914). *The analysis of sensations, and the relation of the physical to the psychical*. Open Court Publishing Company. https://doi.org/10.2307/3604840

Mach, E. (1959). *The analysis of sensations and the relation of the physical to the psychical* (S. Waterlow, Ed., 5th ed., C. M. Williams, Trans.). Dover Publications.

MacKay, D. (2003). *Information theory, inference and learning algorithms*. Cambridge University Press.

Malach, R., Reppas, J. B., Benson, R. R., Kwong, K. K., Jiang, H., Kennedy, W. A.., Ledden, P. J., Brady, T. J., Rosen, B. R., & Tootell, R. B. (1995). Object-related activity revealed by functional magnetic resonance imaging in human occipital cortex. *Proceedings of the National Academy of Sciences of the United States of America*, *92*(18), 8135–8139. https://doi.org/10.1073/pnas.92.18.8135

Mamassian, P., & Landy, M. S. (1998). Observer biases in the 3D interpretation of line drawings. *Vision Research*, *38*(18), 2817–2832. https://doi.org/10.1016/S0042-6989(97)00438-0

Mendola, J. D., Dale, A. M., Fischl, B., Liu, A. K., & Tootell, R. B. (1999). The representation of illusory and real contours in human cortical visual areas revealed by functional magnetic resonance imaging. *The Journal of Neuroscience*, *19*(19), 8560–8572. https://doi.org/10.1523/JNEUROSCI.19-19-08560.1999

Metzger, W. (1941). *Psychologie: Die entwicklung ihrer grundannhamen seit der einführung des experiments*. Steinkopff. https://doi.org/10.1007/978-3-642-53395-2

Metzger, W. (1963). *Psycologie*. Steinkopff Verlag. https://doi.org/10.1007/978-3-662-43189-4

Metzger, W. (1975). *Gesetze des sehens*. Kramer.

Meyer, G. E., & Garges, C. (1979). Subjective contours and the Poggendorff illusion. *Perception & Psychophysics*, *26*(4), 302–304. https://doi.org/10.3758/BF03199884

Meyer, G. E., & Phillips, D. (1980). Faces, vases, subjective contours, and the McCollough effect. *Perception*, *9*(5), 603–606. https://doi.org/10.1068/p090603

Myung, I. J., Forster, M. R., & Browne, M. W. (2000). Guest editors' introduction. *Journal of Mathematical Psychology*, *44*(1), 1–2. https://doi.org/10.1006/jmps.1999.1273

Nakayama, K., & Shimojo, S. (1990). Toward a neural understanding of visual surface representation. *Cold Spring Harbor Symposia on Quantitative Biology*, *55*, 911–924. https://doi.org/10.1101/SQB.1990.055.01.085

Pastore, N. (1971). *Selective history of theories of visual perception, 1650–1950*. Oxford University Press.

Penrose, L. S., & Penrose, R. (1958). Impossible objects: A special type of visual illusion. *British Journal of Psychology*, *49*(1), 31–33. https://doi.org/10.1111/j.2044-8295.1958.tb00634.x

Peterhans, E., & von der Heydt, R. (1987). The role of end-stopped receptive fields in contour perception. In N. Elsner & O. Creutzfeldt (Eds.), *New Frontiers in brain research: Proceedings of the 15th Göttingen neurobiology conference* (pp. 111–127). Thieme Publishing Group.

Peterson, M. A., & Gibson, B. S. (1993). Shape recognition inputs to figure-ground organization in three-dimensional displays. *Cognitive Psychology*, *25*(3), 383–429. https://doi.org/10.1006/cogp.1993.1010

Peterson, M. A., & Gibson, B. S. (1994). Object recognition contributions to figure-ground organization: Operations on outlines and subjective contours. *Perception & Psychophysics*, *56*(5), 551–564. https://doi.org/10.3758/BF03206951

Peterson, M. A., Harvey, E. M., & Weidenbacher, H. J. (1991). Shape recognition contributions to figure-ground reversal: Which route counts? *Journal of Experimental Psychology: Human Perception and Performance*, *17*(4), 1075–1089. https://doi.org/10.1037/0096-1523.17.4.1075

Pinna, B. (1990). *Il dubbio sull'apparire*. Upsel.

Pinna, B. (1991). Anomalous contours and illusion of angularity: Phenomenal and theoretical comparisons. *Perception*, *20*(2), 207–218. https://doi.org/10.1068/p200207

Pinna, B. (1996). Superfici anomale tra l'incompletezza e la completezza, il locale ed il globale. In *Atti* *del Congresso Nazionale della sezione di Psicologia Sperimentale* (pp. 218–220). Capri.

Pinna, B. (2011). Illusory contours and neon color spreading reconsidered in the light of Petter's rule. In L. Albertazzi, G. van Tonder, & D. Vishwanath (Eds.), *Information in perception* (pp. 261–284). MIT Press.

Pinna, B., Ehrenstein, W. H., & Spillmann, L. (2004). Illusory contours and surfaces without amodal completion and depth stratification. *Vision Research*, *44*(16), 1851–1855. https://doi.org/10.1016/j.visres.2004.02.013

Pinna, B., & Grossberg, S. (2006). Logic and phenomenology of incompleteness in illusory figures: New cases and hypotheses. *Psychofenia*, *9*(15), 93–135. https://siba-ese.unisalento.it/index.php/psychofenia/article/download/i17201632vIXn15p93/3265

Pinna, B., Spillmann, L., & Werner, J. S. (2003). Anomalous induction of brightness and surface qualities: A new illusion due to radial lines and chromatic rings. *Perception*, *32*(11), 1289–1305. https://doi.org/10.1068/p3475

Pomerantz, J. R., Goldberg, D. M., Golder, P. S., & Tetewsky, S. (1981). Subjective contours can facilitate performance in a reaction-time task. *Perception & Psychophysics*, *29*(6), 605–611. https://doi.org/10.3758/BF03207379

Pomerantz, J. R., & Kubovy, M. (1986). Theoretical approaches to perceptual organization: Simplicity and likelihood principles. In K. R. Boff, L. Kaufman, & J. P. Thomas (Eds.), *Handbook of Perception and human performance: Vol. 2. Cognitive processes and performance* (pp. 1–46). Wiley.

Purghè, F. (1990). Le superfici anomale e il modello di Kanizsa. Il completamento amodale è realmente un fattore necessario? *Rivista di Psicologia*, *75*(1), 9–24. https://www.poligrafo.it/rivista-di-psicologia-ns-lxxv-1-gennaio-aprile-1990

Raizada, R. D., & Grossberg, S. (2003). Towards a theory of the laminar architecture of cerebral cortex: Computational clues from the visual system. *Cerebral Cortex*, *13*(1), 100–113. https://doi.org/10.1093/cercor/13.1.100

Ramachandran, V. S. (1985). Apparent motion of subjective surfaces. *Perception*, *14*(2), 127–134. https://doi.org/10.1068/p140127

Restle, F. (1970). Theory of serial pattern learning: Structural trees. *Psychological Review*, *77*(6), 481–495. https://doi.org/10.1037/h0029964

Restle, F. (1979). Coding theory of the perception of motion configurations. *Psychological Review*, *86*(1), 1–24. https://doi.org/10.1037/0033-295X.86.1.1

Reynolds, R. I. (1981). Perception of an illusory contour as a function of processing time. *Perception*, *10*(1), 107–115. https://doi.org/10.1068/p100107

Rissanen, J. (1978). Modeling by shortest data description. *Automatica*, *14*(5), 465–471. https://doi.org/10.1016/0005-1098(78)90005-5

Rock, I. (1983). *The logic of perception*. MIT Press.

Rock, I. (1987). A problem-solving approach to illusory contours. In S. Petry & G. E. Meyer (Eds.), *The perception of illusory contours* (pp. 62–70). Springer. https://doi.org/10.1007/978-1-4612-4760-9_6

Rubin, E. (1915). *Synsoplevede Figurer*. Glydendalske.

Rubin, E. (1921). *Visuell wahrgenommene figuren*. Gyldendalske Boghandel.

Sasaki, Y., & Watanabe, T. (2004). The primary visual cortex fills in color. *Proceedings of the National Academy of Sciences of the United States of America*, *101*(52), 18251–18256. https://doi.org/10.1073/pnas.0406293102

Schumann, F. (1900). Beiträge zur analyse der gesichtswahrnehmungen. Erste abhandlung. einige beobachtungen über die zusammenfassung von gesichtseindrücken zu einheiten. *Zeitschrift Für Psychologie Und Physiologie Der Sinnesorgane*, *23*, 1–32. https://echo.mpiwg-berlin.mpg.de/MPIWG:2ACU71YA

Sigman, E., & Rock, I. (1974). Stroboscopic movement based on perceptual intelligence. *Perception*, *3*(1), 9–28. https://doi.org/10.1068/p030009

Simon, H. A. (1972). Complexity and the representation of patterned sequences of symbols. *Psychological Review*, *79*(5), 369–382. https://doi.org/10.1037/h0033118

Simon, H. A., & Kotovsky, K. (1963). Human acquisition of concepts for sequential patterns. *Psychological Review*, *70*(6), 534–546. https://doi.org/10.1037/h0043901

Smith, A. T., & Over, R. (1976). Color-selective tilt aftereffects with subjective contours. *Perception & Psychophysics*, *20*(4), 305–308. https://doi.org/10.3758/BF03199457

Smith, A. T., & Over, R. (1979). Motion aftereffect with subjective contours. *Perception & Psychophysics*, *25*(2), 95–98. https://doi.org/10.3758/BF03198792

Solomonoff, R. J. (1964a). A formal theory of inductive inference. Part I. *Information and Control*, *7*(1), 1–22. https://doi.org/10.1016/S0019-9958(64)90223-2

Solomonoff, R. J. (1964b). A formal theory of inductive inference. Part II. *Information and Control*, *7*(2), 224–254. https://doi.org/10.1016/S0019-9958(64)90131-7

Vitz, P. C., & Todd, T. C. (1969). A coded element model of the perceptual processing of sequential stimuli. *Psychological Review*, *76*(5), 433–449. https://doi.org/10.1037/h0028113

von der Heydt, R., Peterhans, E., & Baumgartner, G. (1984). Illusory contours and cortical neuron responses. *Science*, *224*(4654), 1260–1262. https://doi.org/10.1126/science.6539501

von Grünau, M. W. (1979). The involvement of illusory contours in stroboscopic motion. *Perception & Psychophysics*, *25*(3), 205–208. https://doi.org/10.3758/BF03202987

Von Helmholtz, H. (1867). *Handbuch der physiologischen optik: Mit 213 in den text eingedruckten holzschnitten und 11 tafeln*. Leopold Voss.

Wagemans, J., Elder, J. H., Kubovy, M., Palmer, S. E., Peterson, M. A., Singh, M., & von der Heydt, R. (2012a). A century of Gestalt psychology in visual perception: I. Perceptual grouping and figure-ground organization. *Psychological Bulletin*, *138*(6), 1172–1217. https://doi.org/10.1037/a0029333

Wagemans, J., Feldman, J., Gepshtein, S., Kimchi, R., Pomerantz, J. R., van der Helm, P. A., & van Leeuwen, C. (2012b). A century of Gestalt psychology in visual perception: II. Conceptual and theoretical foundations. *Psychological Bulletin*, *138*(6), 1218–1252. https://doi.org/10.1037/a0029334

Weiss, Y., & Adelson, E. H. (1998). *Slow and smooth: A Bayesian theory for the combination of local motion signals in human vision* (AI Memo No.1624, pp. 1–42). Massachusetts Institute of Technology Intelligence Laboratory. https://hdl.handle.net/1721.1/7252

Weisstein, N., Matthews, M. L., & Berbaum, K. (1974). Illusory contours can mask real contours. *Bulletin of the Psychonomic Society*, *4*(4), 266. https://link.springer.com/journal/40631/volumes-and-issues

Wertheimer, M. (1923). Untersuchungen zur Lehre von Der gestalt. II. *Psychologische Forschung*, *4*(1), 301–350. https://doi.org/10.1007/BF00410640

Zhou, H., Friedman, H. S., & von der Heydt, R. (2000). Coding of border ownership in monkey visual cortex. *The Journal of Neuroscience*, *20*(17), 6594–6611. https://doi.org/10.1523/JNEUROSCI.20-17-06594.2000

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